

## **SPECIAL ISSUE**

# **MULTISCALE MECHANICAL MODELING OF COMPLEX MATERIALS AND ENGINEERING APPLICATIONS 2**

*Guest Editors*

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### **PREFACE**

The present volume is a special issue of selected papers from the second edition of a special symposium session on *Multiscale Mechanical Modelling of Complex Materials and Engineering Applications*, organized within the framework of the *6th International Conference on Processing & Manufacturing of Advanced Materials (THERMEC'2009)*, held in Berlin, Germany, in August 2009.

The early focus of the symposium was to bridge the gap between solid mechanics and material science, providing a forum for the presentation of fundamental, theoretical, experimental, and practical aspects of mechanical modelling of materials with complex microstructures and complex behaviour. This volume follows the issues already edited in connection with the *THERMEC 2006* conference of the same symposium session held in Vancouver, Canada, in July 2006.<sup>1</sup> Each contribution has undergone a standard review process, and only papers that received positive reviews by at least two international referees have been included.

The purpose of this special issue is to give an overview of interdisciplinary research on mechanical modelling of complex materials with an eye to engineering applications, and with emphasis on advanced mathematical tools and computational strategies developed for solution of related initial-boundary value problems. The papers provide a survey of multiscale approaches proposed to describe materials with internal structure, mainly focusing on two-scale homogenization approaches within the (linear or non-linear) elastic framework, also accounting for non-linear complex behaviors, such as damage. The main thread relating these papers can be recognized in the formulation of constitutive models for materials with internal lengths in such a way as to account for any kind of non-locality. This is done either implicitly, as in the case of continua with additional degrees of freedom, or explicitly through the introduction of integral non-local variables.

The paper by K. Sab proposes a procedure for the homogenization of out-of-plane loaded plates made of a linear elastic material with a statistically uniform random microstructure. This procedure consists in a generalization of the homogenization method, already investigated by the author, for periodic plates and random three-dimensional (3D) materials. Starting at the periodic cell problem, to determine the bending and membranal stiffness of the random plate, the author introduces different boundary conditions on a representative volume element (RVE): kinematic or static uniform conditions and periodic conditions. Moreover, statistically invariant stress and strain fields in an infinite plate are considered. These different conditions allow him to derive hierarchical bounds and to acknowledge that, like

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<sup>1</sup>Trovalusci, P., Multiscale mechanical modeling of complex materials and engineering applications, *special issue, in International Journal for Multiscale Computational Engineering*, 5(2), 2007.

for periodic and random 3D materials, periodic boundary conditions on the RVE provide more accurate estimations for the asymptotic homogenized properties.

Working in the conceptual framework of the matched asymptotic expansion for homogenization of hyperelastic materials with many heterogeneities concentrated near a surface, G. Geymonat, F. Krasuckiy, S. Hendili, and M. Vidrascu design and implement a new method for the class of structures with a thin layer of periodically distributed micro-holes. This method is also valid for various kinds of heterogeneities, provided that these are essentially orthogonal to the layer. The approximate solution of the elastic problem is searched within a subdomain containing a layer with vanishing width (reduced to a surface in three dimensions or a line in two dimensions). Satisfactory approximations of the real displacement and stress fields are obtained through two asymptotic expansions, near and far from the layer, respectively (inner–outer expansions). With regard to the isotropic case with a spherical cavity, it is shown that the near-field solution is well reproduced using the outer expansion, which takes into account the matching conditions only quite far from the heterogeneity. The method proves effective for finite-element (FE) numerical simulations due to the robustness with respect to the variation of the dimension of the domain; the geometrical parameters defining the region near the discontinuity and the mesh size.

The paper by C. Chesnais, S. Hans, and C. Boutin, deals with the asymptotic homogenization of periodic discrete structures aimed at describing the macroscopic dynamic behavior of unbraced framed cells. The analysis here is focused on the investigation of atypical gyration modes arising when the transversal inertia balance depends on an independent “inner” bending stiffness, added to the standard bending stiffness, as well as on the shear stiffness. The proposed generalized beam model can be seen as a Timoshenko beam with an internal microshear variable that implicitly satisfies the local balance with the microcouple related to the additional inner inertia. The gyration modes appear at transverse wavelength of the order of the beam section size and cannot be detected with classical beam modelling. Even if no experimental evidence of such gyration modes can be recognized in the literature, the numerical analyses performed for two structures with different inner morphologies and mechanical characteristics confirm the theoretical results.

The aim of the paper by M. L. De Bellis and D. Addessi is to develop a computational homogenization procedure for periodic masonry to apply it to the analysis of in-plane loaded masonry walls. The material is described as a classical continuum at the microscopic level and as a micropolar continuum at the macroscopic level. The theoretical framework of the adopted technique follows the approach originally proposed by Forest and Sab (1998) enriched by the presence of isotropic damage microscopic constitutive laws. Within the framework of non-classical multiscale computational homogenization, higher-order polynomial developments are used to impose periodic displacements on the unit cell boundary. The authors pay attention to the drawbacks arising when such non-homogeneous boundary conditions are assumed. In particular, the presence of non-vanishing terms related to the Cosserat deformation modes result in moduli stiffer than expected. The efficiency of the method is illustrated both in the linear elastic and in the non-linear damaged case with the aid of some numerical results.

The study of the elastic in-plane behavior of periodic masonry made of deformable bricks, much stiffer than mortar, is also addressed in the paper by A. Bacigalupo and L. Gambarotta. In order to properly account for the brick size, the theory of homogenization for special classes of non-classical models of materials (couple-stress, micropolar, second gradient) is considered. A numerical implementation of both classical and micromorphic models, as well as direct FE calculations are presented. The comparison of results concerns a boundary shear layer problem, representative of a masonry wall, with bricks of two different orientations, subjected to a uniform horizontal displacement on the top side. The influence of the gradient of the shear strain with respect to the micropolar curvature is also pointed out and discussed. A comparison of several models shows that, especially in the case of a horizontal brick layer orientation, the non-local effects are best represented by the second gradient approach.

In the work of M. Di Paola and M. Zingales, a non-local elastic continuum model based on fractional operators, presented by the same authors in a previous paper, is extended to the 3D case. The model aims at developing a new fractal differential scheme to solve the mixed boundary value problem of electrostatics in the presence of internal body force with a non-local character. The constitutive function for such a long-range interaction force decays with the fractional power law of distance, which requires the definition of a higher-dimensional fractal operator. With reference to a plane stress test problem for a circular domain of finite radius under in-plane axial symmetric tractions, a numerical solution is obtained with a generalized finite fractional difference scheme. It is shown that the non-local

stiffer elastic response is related to the decaying material parameter accounting for the microstructural effects of the system.

The last work of P. Trovalusci and V. Varano is based on the formulation of a continuum model with additional degrees of freedom (multifield continuum) for the study of the mechanical behavior of composite materials with distribution of microcracks. This model, presented in some earlier papers, has been obtained starting from the kinematics of a complex lattice model and by linking two different material scales via an energy equivalence criterion. Here, a more general, kinematically non-linear, theoretical framework of the non-simple continuous model adopted is given and a further step is made by comparing explicit multifield solutions obtained for a microcracked bar with the numerical solutions of FE simulations for a linear elastic strip with voids. The capability of the linear elastic multifield model in representing the non-linear response of the damaged material and the physical meaning of coupling terms between classical and additional variables is discussed.

We hope these research issues will provide opportunities to identify and discuss future developments in the field of multiscale modelling and design of advanced materials. Our special thanks go to the editor, Jacob L. Fish, for inviting us to select and collect papers in a special issue of this journal and for assisting us in the editing work.

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