DROPWISE CONDENSATION OF METAL VAPORS UNDERNEATH INCLINED SUBSTRATES

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Dropwise condensation of metallic vapor underneath inclined textured substrates is encountered in specialized applications such as power generation and materials processing. Its long-term sustainability poses several challenges. Quantification of substrate wear rate requires knowledge of local wall shear stress and local heat fluxes. In this context, we propose a mathematical model and validate its predictions for condensation of water vapor. The model computes drop size until instability, fluid flow and heat transfer inside the drop, wall shear stresses, and local and average transport coefficients. The extension of the model to liquid metals is presented. The liquid metals considered are mercury, potassium, and sodium. Analysis at the drop scale shows that the skin friction coefficient is inversely proportional to the Reynolds number and decreases with increasing contact angle. The Nusselt number is practically independent of the Reynolds number, but increases with contact angle. Drop sizes at criticality of fall-off and slide-off underneath horizontal and inclined surfaces are determined from gravitational stability. The condensation model is simulated until a dynamic steady state is reached. Instantaneous condensation patterns of metal vapor underneath horizontal and inclined surfaces are compared with those of water. The model shows good agreement with mercury experiments. In metals the minimum drop size is larger whereas the critical drop size and cycle time of condensation are smaller compared with those for water. For this reason, we obtain a higher time-averaged wall shear stress and heat transfer coefficient. For a horizontal surface, condensation properties depend on the properties of the three fluids. This distinction is markedly smaller on a vertical surface.

KEY WORDS: dropwise condensation, metal vapor, mathematical modeling, wall shear stress, heat transfer coefficient, critical drop size, condensation cycle

1. INTRODUCTION

With breakthroughs in chemical texturing, thin film coatings, and manufacturing techniques for physical texturing, special surfaces can be created on which the condensate is in the form of discrete drops rather than a film. It is thus possible to realize dropwise condensation in a more sustained fashion than what was previously thought possible. The heat transfer coefficient in dropwise condensation is higher than in film mode, particularly for low-conductivity fluids such as water (Rose, 2002; Leach et al., 2006). However, dropwise condensation has limitations related to surface leaching that can damage promoter-layer coatings as well as the physical surface topography of the substrate. New-generation coatings have enabled wider use of dropwise condensation in the industry (Enright et al., 2014) where close control over condensation rates is possible when the surface properties of the substrate are carefully chosen in conjunction with the properties of the condensing fluid.

In dropwise condensation, liquid drops grow first over a surface by direct condensation from the vapor phase at nucleation sites and later by coalescence with neighboring drops (Sikarwar et al., 2012; Carey, 2007). At a certain stage, body forces exceed the retention force due to surface tension, leading to drop sliding and eventual fall-off. Owing to this intermittent sweeping action of droplets, portions of the substrate are re-exposed, and a new cycle of generation, growth, and fall-off commences. The overall process is cyclic (Fig. 1), as observed by various authors.
### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$A$</td>
<td>Surface area ($m^2$)</td>
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<tr>
<td>$C_p$</td>
<td>Specific heat at constant pressure ($J/kg \cdot K$)</td>
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<tr>
<td>$d_b$</td>
<td>Base diameter of drop (m)</td>
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<td>$F$</td>
<td>Force acting on drop (N)</td>
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<td>$h$</td>
<td>Heat transfer coefficient, $q''/(T_{sat} - T_w)$ ($W/m^2 \cdot K$)</td>
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<td>$h_l$</td>
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<td>Thermal conductivity of condensate ($W/m \cdot K$)</td>
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<td>Radius of drop (m)</td>
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<td>$Re$</td>
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<td>Heat transfer coefficient, $q''/(T_{sat} - T_w)$ ($W/m^2 \cdot K$)</td>
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<td>$w/U$</td>
<td>Dimensionless velocity in $z$-direction</td>
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<tr>
<td>$\rho$</td>
<td>Density ($kg/m^3$)</td>
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<td>Surface tension of liquid (N/m)</td>
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<td>Saturation temperature</td>
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<td>Reference temperature taken as saturation temperature at atmospheric pressure (K)</td>
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<td>$T_{w}$</td>
<td>Condensing wall, substrate</td>
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<td>$\Delta T$</td>
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</tbody>
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| $C_f$, $\overline{C_f}$ | Local and average skin friction coefficient, $\tau_w/(1/2)\rho U^2$ |}

(Glicksman, 1972; Tanasawa, 1991; Vemuri and Kim, 2006). The direct impact of this quasi-periodicity is a reduction in average drop size before slide-off or fall-off and an enhancement of the heat transfer coefficient (Corradini, 2003). The condensing cycle of nucleation, growth, and subsequent drop removal at criticality can also be exploited in the distillation of liquid metals under vacuum conditions, with purification targets reaching parts-per-billion levels. Such pure metals are required for making sensors and other specialized devices (Murdoch and Burnet, 1966; Harris and Davenport, 1992a, 1992b; Grulke, 2004; Miura et al., 1987; Bottoni, 2004). Information on wall shear distribution is primarily needed to design the substrate against physical/physico-chemical leaching and to estimate its substrate
FIG. 1: Cycle of individual processes in condensation of metal vapor over an inclined substrate; interrelationships shown are: (1) nucleation and growth of drops at each nucleation site by direct vapor condensation; (2) joint growth by direct condensation and coalescence; (3) growth of large drops only by coalescence and smaller drops by direct condensation; and (4) drop instability and condensate removal from the substrate. Fresh nucleation (1) is initiated at the newly exposed sites.

life cycle. Although the average heat transfer coefficient is relatively higher in the case of metal vapor condensation, there are local regions and time instants where it is low, calling for a detailed analysis. Local heat transfer rates also determine the local wall temperature, which governs physico-chemical degradation. Surface reactivity and alloying of the substrate with the condensing liquid metal are also a source of concern. Further, pressure and temperature fluctuations affect the life of the textured surface (Murdoch and Burnet, 1966; Tanasawa, 1991; El-Genk and Tourniera, 2011). Therefore, when the condensing material is a heavy metal, it is necessary to model the condensation cycle in as much detail as possible.

Specially engineered surfaces that promote dropwise condensation are achieved by micro- and nanoscale texturing using physical, topographical, and chemical process, the latter employing deposition of monolayers of organic molecules and thin films. Long-term sustainability of such textured surfaces is a complex subject because the promoter layer is leached by shear stresses generated by the sliding liquid drops and other associated physico-chemical effects during dropwise condensation.

Literature on surface-leaching due to the motion of liquid drops, metallic or otherwise, is limited (Sikarwar et al., 2013). Studies have considered the critical state of a sessile drop on an inclined surface with a focus on contact angle hysteresis, drop shape, and drop retention (Extrand and Kumagai, 1995; ElSherbini and Jacobi, 2004). Kim et al. (2004) performed experiments for measuring the steady sliding velocity of liquid drops on an inclined surface and reported a scaling law to determine the sliding velocity of a liquid with known wetting characteristics. Using Particle Image Velocimetry (PIV), Sakai and Hashimoto (2007) determined the velocity vector distribution inside a sliding sessile drop. The authors reported that the velocity gradient near the liquid-solid interface is higher than it is elsewhere. This analysis was further used to recognize the slipping and rolling components of the sliding velocity and acceleration of the water drop (Suzuki et al., 2008). Das and Das (2009) used smooth particle hydrodynamics to numerically capture the internal circulation inside a sliding drop. Lister et al. (2010) discussed the formation of a
pinned drop from a thin fluid film on the underside of a rigid surface. The authors studied the possibilities of colliding drops bouncing off each other and the surface during coalescence, thus creating a new instability mechanism for drop removal.

The condensing liquid commonly used in earlier studies was water. Only a few researchers considered dropwise condensation of metal vapor. Bakulin et al. (1967) reported the effect of a noncondensable phase on temperature drop at the liquid-vapor interface in dropwise condensation of sodium, potassium, and lithium. Rose (1972) modified a previously reported theory of dropwise condensation and showed that the degree of subcooling affects heat transfer in mercury, although the effect is less than in water. Necmi and Rose (1977) experimentally measured vapor-to-condensing surface temperature difference and the corresponding heat flux for various vapor pressures during condensation of mercury on a vertical substrate. Niknejad and Rose (1984) compared experimental data on mercury with their own theory of dropwise condensation developed for water and found significant differences in the dropwise condensation behavior of these fluids.

The authors of the present study have developed a mathematical model of dropwise condensation on the underside of horizontal and inclined surfaces (Sikarwar et al., 2011). The model is general enough to deal with a variety of fluids ranging from water to liquid sodium. The condensation process is seen to be quasi-periodic, starting from nucleation and proceeding to growth and coalescence, followed by the movement of unstable drops that initiates fresh nucleation on exposed portions of the substrate. From the distribution of drops of various sizes, the instantaneous surface-averaged heat transfer coefficient and wall shear stress are computed. The largest drop diameter achieved depends on gravitational stability with respect to the interfacial forces at the three-phase contact line. Drop instability controls the periodicity of the condensation process and, as a result, the heat transfer coefficient and wall shear stress. A parametric study was carried out for metallic vapors of mercury, potassium, and sodium as a function of hydrophobicity, contact angle hysteresis, degree of subcooling, and nucleation site density. Although the model is general enough for any inclination, condensation patterns on horizontal and vertical substrates are presented for three liquid metals: mercury, potassium, and sodium.

2. MODEL DEVELOPMENT

As a first step, flow and heat transfer are simulated in the three-dimensional drop geometry of liquid metal sliding on an inclined surface. The aim of this simulation is to correlate the skin friction coefficient and Nusselt number of an isolated sliding drop with the Reynolds number and contact angle as parameters for various low-Prandtl-number fluids. The data generated from the individual drop-level model are used for the second stage of condensation, wherein an ensemble of drops periodically condenses under a surface. The correlations serve as input to the overall condensation model in this second step, which involves capturing the entire condensation cycle. The cycle comprises nucleation, growth, coalescence, slide-off, and fall-off, followed by fresh nucleation. The condensation model at the second step is an extension of the authors’ earlier work (Sikarwar et al., 2011), wherein only water was considered as the condensate. The spatiotemporal drop distributions, available condensation area, and local and average heat transfer coefficients are obtained from the overall model of dropwise condensation.

2.1 Drop Sliding underneath an Inclined Substrate

Numerical simulation of a drop sliding on a surface was presented by the authors elsewhere (Sikarwar et al., 2013). Here, the overall methodology is summarized for liquid metals (Hg, K, and Na).

A schematic of a three-dimensional deformed drop with an advancing angle $\theta_{adv}$ and a receding angle $\theta_{rcd}$ is shown in Fig. 2(a). The drop is deformed, and the difference in angles between the advancing (leading) and receding (trailing) sides is the contact angle hysteresis. The contact angle variation in functional form is related to interfacial forces and gravity, apart from surface texture (contact angles and their hysteresis). In dimensionless form, the Bond number becomes important. Data for contact angles and hysteresis of liquid metal drops on metallic surfaces is scarce. In the present work, the variation of these two angles is adapted from ElSherbini and Jacobi (2006), who presented data for a Bond number less than unity. In the condensation of metal vapors, drops grow with time but are drained away; thus, their average diameter remains small. Based on the largest drop size of the three metal liquids, the Bond
FIG. 2: (a) Schematic of fluid motion and heat transfer at the scale of a single drop; (b) computational domain with tetrahedral elements in an unstructured grid; (c) schematic showing merger of drops and the criteria of coalescence.

number range encountered in the present study is 0.01–0.1. Because a variety of surface preparation techniques are available, it is assumed that the contact angles employed in the present study are indeed realizable in practice. In addition, simulations have been carried out with the contact angle and its hysteresis as parameters. The shape of the drop is taken to be part of a sphere with contact angles varying 80–145°. On inclined surfaces, the drop is deformed and the contact angle hysteresis varies 5–45° for the average contact angle $\theta_{avg} = (\theta_{adv} + \theta_{rcd})/2$ in the range 80–145°. The shape of the deformed drop is obtained by the two-circle approximation (ElSherbini and Jacobi, 2004). The frame of reference for flow and transport calculations is fixed in the liquid drop, with the substrate wall moving relative to it at a constant speed.

The governing equations for flow and heat transfer in a single drop moving over a solid surface are the unsteady, incompressible Navier-Stokes and energy equations. Boundary conditions are prescribed over the drop surface as follows: At the wall, the no-slip condition holds while the wall has a prescribed temperature. The liquid boundary in contact with the vapor is taken to be stress-free. At the free surface, pressure is constant, the normal component of velocity is zero, and the shear stress components are zero. The free surface is also taken to be isothermal at a given distinct vapor saturation temperature; the liquid-vapor interfacial thermal resistance is neglected. The numerical simulation is based on collocated finite-volume discretization of the unsteady three-dimensional Navier-Stokes equations over an unstructured mesh. The volume of a single drop is discretized using tetrahedral elements of nearly equal volumes, as shown in Fig. 2(b). Pressure-velocity coupling is treated using a smoothing pressure correction method that results in a SIMPLE-like algorithm. Convective terms are discretized by a hybrid of simple upwind and central differencing. The diffusion terms are discretized using a second-order central-difference scheme. Geometry-invariant features of the tetrahedral elements are used so that the calculation of gradients at cell faces is simplified using nodal quantities of a particular variable. Nodal quantities, in turn, are calculated as a weighted average of the surrounding cell-centered values (Barth and Jespersen, 1989; Frink et al., 1991). The discretized system of algebraic equations is solved by the stabilized biconjugate gradient method (BiCGStab) with a diagonal preconditioner. The overall solution algorithm is similar to that proposed by Date (2005). Points of difference are related to the use of certain invariant properties of the tetrahedral element, a powerful linear equations solver, and a parallel implementation of the computer program. Iterations in the code are run until convergence of $10^{-7}$ is reached in the residuals. From the velocity and temperature data, wall shear stress and heat transfer rates are determined.

Correlations in terms of the skin friction factor and the Nusselt number for low-Prandtl-number liquids that serve as input to the overall condensation model have been derived as follows:

$$ (\tau_f)_{sd} = 58 \text{Re}^{-0.97} \theta_{avg}^{-1.58} $$

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(Nu)_sd = 18.470^{-1.26}_{avg} \quad \text{(2)}

Here, the drop flow Reynolds number varies 5–1,000 and θ_{avg} is 80–145°. Equations (1) and (2) have been developed for liquid metals (Pr < 0.1). Equation (1) is used to estimate the terminal velocity of a sliding drop, and Eq. (2) is used to estimate the convective heat transfer coefficient during the sliding motion of a drop in an ensemble of drops in dropwise condensation.

2.2 Dropwise Condensation Process

Multiscale modeling of dropwise condensation over textured surfaces was discussed by the authors in earlier work (Sikarwar et al., 2012, 2011), and it was found that condensation at timescales of seconds to minutes is not sensitive to atomic-level processes that fix the initial minimum drop radius at the macroscale. Therefore, atomic-level modeling of condensation is dispensed with and drops formed at the initial nucleation sites are directly assigned a minimum possible stable radius from thermodynamic considerations. In the present model, the growth of drops is simulated from drop embryos that grow at specific nucleation sites while the portion of the surface between the growing drops remains dry. Nucleation sites are randomly distributed on the substrate, whereas nucleation site density is a model parameter. The minimum size of a drop found by thermodynamic considerations is (Carey, 2007)

\[ r_{min} = \left[ \frac{2\sigma_{lv}T_w}{h_{int}l(T_{sat} - T_w)} \right] \quad \text{(3)} \]

At the nucleation sites, drops first grow by direct condensation of vapor and then by coalescence with neighboring drops. When a certain drop size is reached, the body force component (the gravity force in this case) exceeds the surface tension and the drop is set in motion. The growth rate of the drop at each nucleation by direct condensation is limited by the thermal resistance offered by the drop and the substrate in transferring latent heat release to the surrounding atmosphere. The growth rate of the drop radius can be derived as (Sikarwar et al., 2011)

\[ \frac{dr}{dt} = \left( \frac{4\Delta T}{\rho l h_{int}} \right) \left[ \frac{1 - r_{min}}{r} \right] \left[ \frac{2}{h_{int} + \rho l (1 - \cos \theta_{avg})} \right] \left[ \frac{(1 - \cos \theta_{avg})}{(2 - 3 \cos \theta_{avg} + \cos^3 \theta_{avg})} \right] \quad \text{(4)} \]

In Eq. (4), the rate of condensation at each nucleation site is estimated using a quasi-one-dimensional approximation of thermal resistances: the interfacial resistance at the vapor-liquid boundary and conduction resistance through the drop. Drops grow by direct condensation up to a size that is on the order of the distance between adjacent nucleation sites. Beyond this point, coalescence between neighboring drops takes place. Subsequent growth occurs via a combination of direct condensation and coalescence. At the commencement of coalescence, at least two drops touch each other and are replaced by a drop of equal total volume. The new drop is placed at the resultant weighted center of mass. The distance between two nucleation sites \( i \) and \( j \) on the substrate is calculated as

\[ l_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \quad \text{(5)} \]

Coalescence is enforced when

\[ l_{ij} - (r_i + r_j) < 10^{-6} \quad \text{(6)} \]

Two situations—(1) when the contact angles of the coalescing droplets are less than \( \pi/2 \) and (2) when the contact angle is greater than \( \pi/2 \)—are distinct from a coalescence point of view. In the former, the three-phase contact lines of the two coalescing droplets make the first contact to initiate coalescence. In the latter, the first contact is not made at the three-phase contact line [Fig. 2(c)]. However, the coalescence criterion described previously is valid for both cases.

Interfacial energy release during coalescence generates fluid velocity and contributes to viscous dissipation, but it is neglected in the present work. The entire time period of the dropwise condensation cycle, from a virgin surface
to the first fall-off or slide-off, is typically on the order of several minutes. However, our experiments show that drop coalescence happens in a time period on the order of 100 ms or fewer (Somwanshi et al., 2014; Khandakar and Muralidhar, 2013). This timescale, although small, depends on the size of the individual drops and other thermophysical properties such as viscosity, surface tension, contact angles, and surface texture conditions at the operating temperature. With a timescale of 10 ms, the energy release rate is calculated for liquid sodium as $1.74 \times 10^{-3}$ W. The corresponding heat flux (using the base area of the new drop) is 80.8 W/m$^2$. Compared with the average heat flux of 0.1–1 MW/m$^2$ during dropwise condensation (Fig. 3), this is truly small and can be neglected.

Through direct condensation and coalescence, drops are allowed to grow to a critical size when gravity exceeds the retention force of the surface tension. The drop may then fall off for a horizontal substrate or slide over an inclined substrate. Hidden sites underneath the original drop and the swept area generated by moving drops for inclined substrates become active once the droplets fall or slide off, and the entire process is repeated. The critical radius of the drop at slide-off for an inclined substrate has been derived by the authors as (Sikarwar et al., 2011):

$$r_{\text{crit}} = \sqrt{\frac{1.25 \sin \theta_{\text{avg}}}{(2 - 3 \cos \theta_{\text{avg}} + \cos^3 \theta_{\text{avg}})}} (\cos \theta_{\text{rcd}} - \cos \theta_{\text{adv}}) \left(\frac{\sigma}{g \sin \alpha (\rho_l - \rho_v)}\right)$$

Equation (7) shows that the ratio $r_{\text{crit}}$ is a function of substrate orientation $\alpha$, contact angle hysteresis, and surface energy. A simpler form of Eq. (7) can be developed from regression relationship as

$$\frac{r_{\text{crit}}}{r_{\text{cap}}} = f(\Delta \theta, \alpha, \theta_{\text{avg}})$$

Here, $r_{\text{cap}} = \sqrt{\frac{\sigma}{g (\rho_l - \rho_v)}}$ is capillary length scale and distinct for each condensing metal vapor.

When the drop achieves terminal velocity, the sum of all external forces acting on it in a direction parallel to the inclination of the substrate is zero. These forces are the component of weight parallel to the substrate ($F_{g||}$), surface retention force of the deformed drop due to surface tension ($F_r$), and wall friction ($F_s$). Hence,

$$F_{g||} + F_r + F_s = 0$$

![FIG. 3: (a) Comparison of average substrate heat flux plotted as a function of degree of subcooling ($\Delta T$) for dropwise condensation of mercury over a vertical plate at a saturation temperature of 139°C and contact angle hysteresis ($\Delta \theta$) = 22° and 25°. Nucleation site density is taken as $10^6$ per cm$^2$. (b) Validation of the present model with the data adapted from Rose (2002) for condensation of water on a chemically textured vertical surface.](image-url)
Here, \( F_{g||} \) is the component of weight parallel to the inclined substrate, \( F_r \) is the retention force opposing the drop motion, and \( F_s \) is the viscous force owing to the relative velocity between the fluid and the substrate. These force components are respectively written as

\[
F_{g||} = \frac{\pi}{3 \sin^3 \theta_{avg}} r_b^3 \left( 2 - 3 \cos \theta_{avg} + \cos^3 \theta_{avg} \right) \rho_l g \sin \alpha 
\]

(9b)

\[
F_r = 2 \int_0^\pi \sigma \cos \theta \cos \varphi r_b d\varphi 
\]

(9c)

\[
F_s = (C_f)_{sd} \left( \frac{1}{2} \rho_l U^2 \right) A_b 
\]

(9d)

The azimuthal angle over the base of the drop is denoted \( \varphi \), and \( (C_f)_{sd} \) is the average skin friction coefficient determined at the scale of the drop by solving the fluid flow equations (Eq. 1). The terminal velocity can now be calculated as

\[
U = \sqrt{\frac{2(F_{g||} - F_r)}{(C_f)_{sd} \rho A_b}} 
\]

(9e)

The sliding drop wipes off other drops that lie in its path, exposing a fresh area for renucleation. In addition, the mass and volume of the drop increase during its passage over the surface because of the incorporation of smaller drops. The sliding velocity of the drop then needs to be recalculated from Eq. (9e). As the drop size increases during sliding, it may also reach fall-off criticality at an intermediate time instant. The fall-off criterion can be derived by examining the sum of all forces acting on the drop in the vertical direction. Accordingly, the drop leaves the surface when its radius exceeds (Sikarwar et al., 2011)

\[
r_{max} = \sqrt{\left[ 3 (\sin \theta_{avg}) (\sin \theta_{rcd} + \sin \theta_{adv}) \right] \left[ \frac{\sigma}{(g \cos \alpha) (\rho_l - \rho_v)} \right]} 
\]

(10)

Heat transfer during dropwise condensation is calculated by determining the rate of condensation at the free surface of individual drops located underneath the substrate. The gaps between the growing drops are assumed to be inactive for heat transfer. The heat transfer rate \( q \) is a function of nucleation site density and drop radius growth rate at each nucleation site. It can be determined as follows. The mass of condensate accumulated at the \( i \)th nucleation site over a time interval \( \Delta t \) is calculated as

\[
(\Delta m)_i = \rho \frac{\pi}{3} \left( 2 - 3 \cos \theta_{avg} + \cos^3 \theta_{avg} \right) (r_{new}^3 - r_{old}^3)_i 
\]

(11)

with \( N \), the number of active nucleation sites at a given time step, the total quantity of condensate is obtained:

\[
\Delta m = \sum_{i=1}^N (\Delta m)_i 
\]

(12)

Therefore, the average rate of condensation underneath a given substrate over a time period \( t \) is

\[
(m)_{avg} = \frac{\sum_{j=1}^K (\Delta m)_j}{t} \quad \text{where,} \quad t = \frac{\sum_{j=1}^K (\Delta t)_j}{K} 
\]

(13)

Here, \( t \) is the time period of condensation and \( K = (t/\Delta t) \) is the number of time steps. The average heat transfer coefficient over an area (\( A \)) of the substrate during dropwise condensation is

\[
h = \frac{(m)_{avg} h_{lv}}{A \Delta T} 
\]

(14)
The formulation for determining the net surface shear stress acting on the condensing surface due to continuous drop sliding is described next. Shear forces are generated by each drop when it begins sliding after achieving criticality. Thereafter, the mass of the droplet may increase during its travel on the substrate and so the shear force is generated by each moving drop. This necessitates averaging in space and time to obtain the average due to the movement of a droplet ensemble. The average shear stress on the substrate is defined as

$$
\bar{\tau}_s = \frac{\bar{F}_s}{A} = \frac{1}{A} \left[ \sum_{j=1}^{K} \sum_{i=1}^{N} \left[ F_{ji} \right] t \right]
$$

(15)

The average skin friction coefficient on the substrate due to the drop ensemble can then be estimated as

$$
\bar{C}_f = \left( \frac{\bar{\tau}_s}{\frac{1}{2} \rho U_{rep}^2} \right) \text{, where } U_{rep} = \sqrt{g \cdot r_{crit}}
$$

(16)

Although no droplet motion can commence unless the radius exceeds $r_{crit}$ (either by individual droplet growth by direct condensation or by coalescence), the ensuing terminal velocity for moving drops may be different depending on the path traversed and the amount of additional mass gathered, because there are other droplets in the way. Therefore, in Eq. (16) the representative drop velocity is generically obtained by scaling gravitational acceleration against net kinetic energy of the droplet. The physico-chemical characteristics of the liquid-solid combination are embedded in the critical radius. Thermophysical properties of the vapor and liquid phases are evaluated at the average of the substrate and saturation temperatures.

3. VALIDATION

Validation of the coupled Navier-Stokes and energy equations solver was discussed by the authors in detail elsewhere (Sikarwar et al., 2013). The dropwise condensation model was validated for water in a previous study (Sikarwar et al., 2011) in terms of condensation patterns, area coverage, and cycle times. The condensation model described in the previous section for liquid metals is validated in terms of heat fluxes for condensation of mercury vapor on a vertical surface. The experimental data for the surface-averaged wall heat flux as a function of vapor-to-surface temperature difference is adopted from the work of Necmi and Rose (1977) and Niknejad and Rose (1984). These researchers did not provide the contact angle hysteresis for mercury on steel, and it is not available in the literature. Hence, validation has been carried out for a contact angle hysteresis in the range 22–24°. The thermophysical properties of mercury are summarized in Table 2. The nucleation site density used for validation is $10^6$ per cm$^2$. Figure 3(a) compares the present model and the experiments of Necmi and Rose (1977). It is seen that the data relating heat flux and degree of subcooling are weakly sensitive to contact angle hysteresis. For a contact angle hysteresis of 22°, the figure shows good agreement for smaller levels of subcooling. The agreement for higher temperature differences is excellent for a hysteresis of 24°. Considering the fact that the numerical data are surface- and cycle-averaged, the comparison of the model and experiments can be considered quite good. The mathematical model was also validated for water vapor condensation experiments (Rose, 2002). With a choice of average contact angle =100°, hysteresis =18°, and nucleation site density =10$^6$ cm$^{-2}$, Fig. 3(b) shows that the agreement between the surface heat transfer data as a function of subcooling and the saturation temperature is good as well.

4. NUCLEATION SITE DENSITY

For liquid metals, the size of the smallest drop ($r_{min}$) initially formed on the substrate [Eq. (3)] is on the order of a few nanometers. Validation of this initial length scale of the embryo and the timescale required for its formation has not been carried out in this work. In addition, there is considerable uncertainty in the literature regarding the prescription of nucleation site density ($N$), which is strongly influenced by the thermophysical properties of the condensing fluid, the physico-chemical properties of the substrate, the degree of subcooling, and the substrate morphology. Values
from the literature range $10^5$–$10^8$ cm$^{-2}$ (Leach et al., 2006). Rose (1976) deduced an equation of the form $N = 0.037/r_{\text{min}}^2$, with which the nucleation site densities were calculated as $5.9 \times 10^9$ and $2.9 \times 10^{11}$ cm$^{-2}$.

To gauge the sensitivity of heat transfer rates to $N$, a parametric study is presented for the three liquid metals, with nucleation site density varying $10^4$–$10^9$ cm$^{-2}$. The variation in the heat transfer coefficient with $N$ is shown in Figs. 4(a)–4(c), where the degree of subcooling is included as an important parameter. The heat transfer coefficient increases with nucleation site density for all degrees of subcooling. This trend is understandable because a large density leads to a reduction in drop size before coalescence. Early coalescence creates a virgin space for the formation of new drops, thus increasing the population of small drops. A high nucleation site density takes condensation toward a combined condensation-coalescence regime. Because the conduction resistance of a small drop is small, the overall heat transfer coefficient is substantially higher for a large nucleation site density. Figure 4(d) is a revised plot in which the effect of degree of subcooling is combined with that of the heat transfer coefficient to produce a unique dependence on $N$. For all three metals, the composite function increases with $N$; in addition, the three curves are sufficiently differentiated.

In view of a clear dependence of heat transfer on nucleation site density for each metal, an initial value of $N = 10^6$ cm$^{-2}$ has been adopted in the following sections.

**FIG. 4:** Average wall heat flux as a function of subcooling with nucleation site density as a parameter: (a) mercury, (b) potassium, and (c) sodium. (d) Correlation of heat transfer coefficient (MW/m$^2$·K) scaled by degree of subcooling with nucleation site density for a saturation temperature of 600 K.
5. RESULTS AND DISCUSSION

The results of the present study are reported in two parts: (1) flow and temperature distribution inside an isolated sliding drop and (2) simulation of the complete dropwise condensation process over textured surfaces. The metal vapors of mercury, potassium, and sodium are considered. In the first part, flow and transport are taken to be steady. In the second part, the simulation is essentially unsteady and continues until a dynamic steady state is reached. As noted earlier, Tables 1 and 2 summarize the parameters and thermophysical properties used for simulation.

5.1 Velocity and Temperature Distribution in a Single Drop

Figure 5 shows the streamlines and dimensionless velocity distribution in a drop at Reynolds numbers of 10 and 100 for liquid mercury, potassium, and sodium on a frontal plane (z = 0). For the plane selected, the surface motion is to the right, in the plane of the paper. The drop shape has been generated for an average contact angle of 105° and advancing and receding angles of 113° and 97°, respectively. The relevant thermophysical properties of the three liquid metals are evaluated at the average temperature of the vapor and the substrate. Owing to differences in advancing and receding angles as well as surface tension, minor differences in drop shape are to be seen among the three fluids. Accordingly, the flow distributions, although quite similar, show minor differences. For a higher

TABLE 1: Parameters used for the pictorial views of dropwise condensation in Figs. 11–16

<table>
<thead>
<tr>
<th>Liquid metal</th>
<th>$T_s$ (K)</th>
<th>$T_w$ (K)</th>
<th>$\sigma$ (N/m)</th>
<th>$\alpha$ (deg)</th>
<th>$\theta_{avg}$ (deg)</th>
<th>$\theta_{rcd}$ (deg)</th>
<th>$\theta_{adv}$ (deg)</th>
<th>$\Delta \theta$ (deg)</th>
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</thead>
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<td>599</td>
<td>0.42</td>
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<td>0</td>
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<td>90</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>90</td>
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<td>10</td>
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<td>599</td>
<td>0.09</td>
<td>0</td>
<td>0</td>
<td>105</td>
<td>90</td>
<td>105</td>
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<td></td>
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<td>90</td>
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</tr>
<tr>
<td>Na</td>
<td>600</td>
<td>599</td>
<td>0.17</td>
<td>0</td>
<td>0</td>
<td>115</td>
<td>90</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>109</td>
<td>121</td>
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</tr>
<tr>
<td>Water</td>
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<td>300</td>
<td>0.073</td>
<td>0</td>
<td>0</td>
<td>90</td>
<td>96.5</td>
<td>86</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>107</td>
<td>107</td>
<td>20</td>
</tr>
</tbody>
</table>

TABLE 2: Thermodynamic and thermophysical properties used in the simulation of metal vapor condensation at various saturation temperatures

<table>
<thead>
<tr>
<th>Liquid metal</th>
<th>$T_s$ (K)</th>
<th>$P_{sat}$ (N/m²)</th>
<th>$\rho_v$ (kg/m³)</th>
<th>$C_p$ (J/kg · K)</th>
<th>$k$ (W/m · K)</th>
<th>$\mu$ (Pa · s)</th>
<th>$h_{le}$ (kJ/kg)</th>
<th>$\sigma$ (N/m)</th>
<th>$Pr$ —</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hg</td>
<td>350</td>
<td>9.81E+00</td>
<td>13407</td>
<td>6.75E-04</td>
<td>137.7</td>
<td>9.18</td>
<td>304.1</td>
<td>0.001309</td>
<td>304.1</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>1.40E+02</td>
<td>13287</td>
<td>8.47E-03</td>
<td>136.5</td>
<td>9.8</td>
<td>302.4</td>
<td>0.001171</td>
<td>302.4</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>5.65E+04</td>
<td>12809</td>
<td>2.25E+00</td>
<td>135.5</td>
<td>11.95</td>
<td>295.9</td>
<td>0.000911</td>
<td>295.9</td>
</tr>
<tr>
<td>632*</td>
<td>1.032E+05</td>
<td>12700</td>
<td>3.77E+00</td>
<td>136.1</td>
<td>12.2</td>
<td>0.000885</td>
<td>295.00</td>
<td>0.000885</td>
<td>295.00</td>
</tr>
<tr>
<td>K</td>
<td>400</td>
<td>9.20E+01</td>
<td>13287</td>
<td>8.47E-03</td>
<td>136.5</td>
<td>9.8</td>
<td>302.4</td>
<td>0.001171</td>
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<td>600</td>
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<td>11.95</td>
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</tr>
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<td>12700</td>
<td>3.77E+00</td>
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<td>0.000885</td>
<td>295.00</td>
<td>0.000885</td>
<td>295.00</td>
</tr>
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<td>1,032*</td>
<td>1.032E+05</td>
<td>664</td>
<td>4.95E-01</td>
<td>750.0</td>
<td>39.3</td>
<td>1.92</td>
<td>1920</td>
<td>0.000885</td>
<td>1920</td>
</tr>
<tr>
<td></td>
<td>700</td>
<td>1.11E-02</td>
<td>850</td>
<td>4.46E-04</td>
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<td>71.5</td>
<td>2.66E-04</td>
<td>2.66E-04</td>
<td>2.66E-04</td>
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<tr>
<td>850*</td>
<td>1.29E+03</td>
<td>814</td>
<td>7.80E-03</td>
<td>1.28E+03</td>
<td>64.6</td>
<td>2.16E-04</td>
<td>4.190</td>
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<td>4.06E-03</td>
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<tr>
<td>1,150*</td>
<td>1.032E+05</td>
<td>742</td>
<td>2.78E-01</td>
<td>1.28E+03</td>
<td>52.3</td>
<td>1.54E-04</td>
<td>3.890</td>
<td>0.115</td>
<td>3.77E-03</td>
</tr>
</tbody>
</table>

*Reference values used to formulate the dimensionless heat transfer correlation for dropwise condensation in Eq. (21).
Streamlines showing motion of the liquid inside the sliding drop; contours of dimensionless velocity at Re = 10 and 100 on the plane considered in (a) for mercury, potassium, and sodium.

Reynolds number, the streamlines are densely spaced near the solid wall. At Re = 100, the velocity magnitude near the free surface is close to the wall speed but in the opposite direction. The wall pressure and shear stress distributions at the base of the drop are shown in Figs. 6 and 7, respectively. Wall pressure and shear stress become quite large near the three-phase contact line of the drop. From the viewpoint of surface wear characteristics, so also from any other associated physico-chemical action, it can be concluded that substrate damage is initiated mostly in the vicinity of the three-phase contact line.

Temperature contours for the three liquids over the central x–y plane are shown in Fig. 8. Apart from drop shape, the differences in the Prandtl numbers of the three liquids contribute to differences in the temperature field. Although the temperature gradients at the free surface and the wall are higher compared with that at the center, clear thermal boundary layers are not formed owing to the large thermal conductivity of liquid metals. Hence, heat transfer between
the wall and the liquid has a dominant diffusional component. Figure 9 shows the local heat transfer coefficient distributed over the base of the drop. Analogous to wall shear stress, large heat transfer rates are realized in the vicinity of the three-phase contact line.

On the basis of the data obtained, correlations of the average skin friction coefficient and the Nusselt number have been developed in the present study, as noted in Eqs. (1) and (2). The drop shape is factored in terms of the average contact angle. The skin friction coefficient is correlated with the Reynolds number and the contact angle as given by Eq. (1). The correlation shows that the skin friction coefficient depends strongly on the average contact angle $\theta_{\text{avg}}$ (radians), being higher for smaller values. The near reciprocal dependence on the Reynolds number shows fully developed flow behavior analogous to internal geometries rather than the boundary-layer flow behavior seen in external flows. The dependence of the average heat transfer coefficient on the Reynolds number is consistently seen to be mild. The correlation for low-Prandtl-number liquids is derived without explicit reference to Reynolds numbers as given by Eq. (2). The sharp dependence of heat transfer on the drop shape is brought out by Eq. (2) through the contact angle. The absence of the Reynolds number in the correlations indirectly eliminates drop size (and speed) from further consideration. This trend is particular to liquid metals and is not applicable to high-Prandtl-number liquids such as water (Tanasawa, 1991). Equations (1) and (2) have a regression coefficient of better than 99.1%.

5.2 Dropwise Condensation: Process Simulation

The dropwise condensation of mercury, potassium, and sodium vapors, respectively, underneath surfaces of various inclinations (0–90°) is simulated using the discussion from Section 2.2. The condensing surface chosen in
the present work does not have any preferred texturing except what promotes dropwise condensation. Among the three condensing fluids, Table 2 shows a substantial variation in latent heat release as well as respective surface tensions.

The goal of process simulation is to obtain the quasi-steady condensation cycle shown schematically in Fig. 1. The intermediate processes in the cycle are nucleation, growth by direct condensation, coalescence, and slide-off/fall-off, to be followed by fresh nucleation at the available sites. The critical size of the drop that is destabilized over an inclined surface is an important parameter in fixing the cycle-averaged heat transfer rate. It is determined from a force balance calculation in a direction parallel to the inclined surface in the direction of inclination. The drop’s nondimensional critical size \( (r_{\text{crit}}/r_{\text{cap}}) \) is a function of the contact angle and its hysteresis as well as the orientation of the substrate [Eq. (7)]. The variation in critical drop size with respect to contact angle hysteresis and substrate orientation for a prescribed average contact angle is shown in Fig. 10(a). The critical drop size decreases with substrate hydrophobicity, causing a reduction in the cycle time, and so results in more frequent instances of renucleation. The reduction in cycle-averaged drop size is an important factor in increasing heat transfer from strongly hydrophobic surfaces. For ease of calculation, the data in Fig. 10 is correlated for the three liquid metals as

\[
\frac{r_{\text{crit}}}{r_{\text{cap}}} = (2.1612 - 0.7699\theta_{\text{avg}}) (\Delta\theta)^{0.5} \alpha^{-0.4266}
\]

The correlation coefficient in Eq. (17) is 99.8%, as shown in Fig. 10(b). The equation simplifies dropwise condensation calculations and considerably reduces computational time.

FIG. 7: Wall shear stress at the base of the drop for \( \text{Re} = 10 \) and 100 for mercury, potassium, and sodium.
The spatiotemporal drop distributions from initial nucleation to instability are shown in Figs. 11–16. They are presented for a wall subcooling of 1 K. For higher subcooling values, the substrate-averaged properties are separately discussed. Figures 11–13 pertain to the underside of a horizontal substrate whereas Figs. 14–16 pertain to condensation over a vertical surface. Based on the complete set of simulation data, certain global attributes of the condensation pattern are summarized in Table 3. These are the minimum drop radii formed at nucleation \( r_{\text{min}} \), the drop radius when instability sets in as slide-off over the surface \( r_{\text{crit}} \), and the drop radius when the drop detaches and falls off \( r_{\text{max}} \). Other quantities of interest are condensing cycle time, average coverage area of the surface by the condensing drops, and wall heat transfer rates.

Table 2 shows that the latent heat release during the condensation of sodium is quite large and hence the minimum radius formed during nucleation is small when compared with the other fluids. As a result, the time taken for the drops to grow on a horizontal surface is quite large for a given nucleation site density (Table 3). The cycle time drastically decreases over a vertical surface and that for all three fluids is comparable. Despite these anomalies in cycle time and minimum drop radius, the three fluids have comparable heat transfer coefficients. Differences arise owing to their thermophysical properties, with sodium having the largest thermal conductivity. Among the three liquid metals, mercury generates large drops at a minimum radius and smaller drops at instability and small cycle times. These factors indicate the possibility of a high heat transfer coefficient. However, this advantage is compensated by a lower thermal conductivity. Table 3 shows that the three metals have similar heat transfer coefficients. With a fluid like water (Pr = 6.5), the largest drop size possible is higher, the cycle time is longer, and the thermal conductivity is lower. These three factors contribute to a considerably lower heat transfer coefficient.

For the three liquid metals, the maximum drop sizes reached are on the order of a few millimeters. Figures 11(b), 12(b), and 13(b) show the available nucleation sites (proportional to the difference between the total substrate area and the area covered by the drops) with respect to time underneath the horizontal substrate. It is seen that the area coverage decreases as drops grow by direct condensation and subsequent coalescence. After passage of the initial transients, dynamic steady state is reached and the area available for condensation fluctuates with time. Underneath
FIG. 9: Distribution of wall heat transfer coefficient (kW/m$^2$ · K) for mercury, potassium, and sodium at Re = 10 and 100.

FIG. 10: (a) Critical base radius of the drop at instability ($r_{\text{crit}}/r_{\text{cap}}$) for a pendant arrangement plotted as a function of substrate orientation ($\alpha$). Contact angle hysteresis is a parameter with the average contact angle at 105°. (b) Parity plot between Eq. (7) and the correlation, Eq. (17), shows an excellent match.
FIG. 11: (a) Spatiotemporal drop distribution during dropwise condensation of mercury vapor at 600 K with subcooling of $\Delta T = 1$ K underneath a horizontal substrate; (b) cyclic variation of available nucleation sites with respect to time; and (c) cyclic variation of the instantaneous heat transfer rate during condensation of mercury vapor.

a horizontal substrate, the number of available nucleation sites is small for mercury as compared with sodium and potassium. The instantaneous heat transfer rates shown in Figs. 11(c), 12(c), and 13(c) clearly depend on available nucleation sites, drop growth rate, and drop size at criticality.
FIG. 12: (a) Spatiotemporal drop distribution during dropwise condensation of potassium vapor at 600 K with subcooling of $\Delta T = 1$ K underneath a horizontal substrate; (b) cyclic variation of nucleation sites with respect to time; and (c) cyclic variation of the instantaneous heat transfer rate during condensation of potassium vapor.

Condensation patterns, temporal variations in available nucleation sites, and surface-averaged heat transfer rates are shown in Figs. 14–16 for a vertical surface. As opposed to a horizontally placed substrate, drop instability leads to a sliding motion over the inclined surface followed by fall-off when the coalesced drop becomes sufficiently heavy.
FIG. 13: (a) Spatiotemporal drop distribution during dropwise condensation of sodium vapor at 600 K with subcooling of $\Delta T = 1$ K underneath a horizontal substrate; (b) cyclic variation of available nucleation sites with respect to time; and (c) cyclic variation of the instantaneous heat transfer rate during condensation of sodium vapor.

Because of the sweeping action of a moving drop over the substrate, sliding frequency, time variation in available nucleation sites, and heat transfer rates are higher for the vertical configuration when compared to the horizontal. The critical size of the sodium drop just before the commencement of sliding is higher because it can transfer a greater heat...
load to the substrate before conduction resistance becomes significant. When compared to mercury and potassium, it continues growing for a longer duration. The larger drops grow primarily through coalescence. Coalescence plays an important role in generating bare areas for renucleation and is clearly captured in the numerical simulation. For
TABLE 3: Summary of results obtained from simulation of metal vapor condensation compared with water

<table>
<thead>
<tr>
<th>Liquid metal</th>
<th>( T_s ) (K)</th>
<th>( \theta_{avg} ) (deg)</th>
<th>Nucleation site density (cm(^{-2}))</th>
<th>( \Delta T ) (K)</th>
<th>( r_{min} ) (( \mu )m)</th>
<th>( r_{cap} ) (mm)</th>
<th>Substrate orientation (mm)</th>
<th>( r_{crit} ) and ( r_{max} ) (s)</th>
<th>Cycle time (%)</th>
<th>Area coverage</th>
<th>Heat transfer coefficient, ( h ) (MW/m(^2) · K)</th>
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</thead>
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<tr>
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<td>0.1234</td>
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<td></td>
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<td>0.54</td>
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<td>Vertical</td>
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<td>818</td>
<td>71</td>
<td>0.107</td>
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</tbody>
</table>

this reason, these factors produce higher heat transfer rates for the substrate during dropwise condensation of metal vapor.

Owing to conduction resistance, the rate of direct condensation per unit area on larger drops is lower than on smaller ones; however, unlike the case of water, for example, the difference is small because of the large thermal conductivity of liquid metals. This trend is reproduced in the numerical simulation and can be seen in Figs. 11–16. Soon after a drop is nucleated, the heat transfer rate is high and diminishes as drop size increases. There are considerable differences within a drop, heat transfer coefficient being very large at the contact line (Fig. 7). The central portion of the drop offers the largest thermal resistance, leading to a minimum heat transfer coefficient [see Bansal et al. (2009)] for an experimental validation of this phenomenon). Immediately circumscribing the contact line of the drop, the re-exposed nucleation sites ensure lower thermal resistance to heat transfer. These factors explain why the cycle of drop formation, growth, and fall-off ultimately enhances the overall heat transfer rate.

As seen in Eq. (17), a large contact angle reduces the size of the drop at criticality, causing a larger population of small drops, on average, over the substrate. Consequently, the heat transfer coefficient is enhanced. To a first approximation, the increase in heat transfer rates can be directly correlated with the departure radius in dropwise condensation, the departure radius being a function of substrate inclination, contact angle, and fluid properties. Heat transfer rate as a function of maximum drop diameter reached before instability is shown in Fig. 17(a). The relationship of heat transfer coefficient of pendant mode condensation with substrate orientation is shown in Fig. 17(b). The following dimensional correlations (with \( h \) in MW/m\(^2\) · K, \( r_{crit} \) in mm, and \( \alpha \) in radians) have been developed from the data in Fig. 17:

Mercury: \( h = 0.28r_{crit}^{-0.7} \)  
Potassium: \( h = 0.8r_{crit}^{-0.51} \)  
Sodium: \( h = 0.9r_{crit}^{-0.46} \)  
Mercury: \( h = 0.49\alpha^{0.24} \)  
Potassium: \( h = 0.65\alpha^{0.21} \)  
Sodium: \( h = 0.79\alpha^{0.14} \)

Eq. sets (18a)–(18c) and (19a)–(19c) are equivalent because \( r_{crit} \) depends on \( \alpha \), as per Fig. 10 and Eq. (17), and can be independently used.

6. SATURATION TEMPERATURE AND DEGREE OF SUBCOOLING

The variation in heat transfer coefficient, averaged over the surface and several cycles of condensation, is shown in Fig. 18 for horizontal and vertical substrates as a function of vapor temperature and degree of subcooling. The data
FIG. 15: (a) Spatiotemporal drop distribution during dropwise condensation of potassium vapor at 600 K with subcooling of $\Delta T = 1$ K over a vertical substrate; (b) cyclic variation of available nucleation sites with respect to time; and (c) cyclic variation of the instantaneous heat transfer rate during condensation of potassium vapor.

show the heat transfer coefficient increasing with metal vapor temperature because of a reduction in latent heat, an increase in drop growth rate and hence drop diameter, early instability, and a corresponding reduction in cycle time. The increase in the heat transfer coefficient of sodium is higher than those of potassium and mercury because of
sodium’s higher thermal conductivity and latent heat. The effect of subcooling is similar because it increases the driving potential of heat transfer and rapidly increases drop size, leading to early instability. These trends are visible in Fig. 19, where the heat transfer coefficient is seen to increase with $\Delta T$.

On the basis of the numerical data, the heat transfer coefficient (kW/m$^2$·K) empirically correlates with the critical radius of the drop (mm), the degree of subcooling (K), and the saturation temperature (K) for the three liquid metals on an inclined plate with a nucleation site density of $10^6$ cm$^{-2}$ as follows:
FIG. 17: Dependence of the heat transfer coefficient: (a) on departing drop radius; (b) on substrate inclination with the horizontal; (c) on degree of subcooling on the vertical substrate; (d) on saturation temperature for a vertical substrate.

Mercury: 
\[ h = (0.6\Delta T + 4) T_{\text{sat}}^{0.64} r_{\text{crit}}^{-0.7} \]  
(20a)

Potassium: 
\[ h = (1.3\Delta T + 5.7) T_{\text{sat}}^{0.75} r_{\text{crit}}^{-0.51} \]  
(20b)

Sodium: 
\[ h = (1.1\Delta T + 6.3) T_{\text{sat}}^{0.78} r_{\text{crit}}^{-0.46} \]  
(20c)

In view of mercury’s thermophysical properties, the dependence of the heat transfer coefficient on saturation temperature and degree of subcooling for it is smaller when compared with those of sodium and potassium.

In a given experiment with one of the condensing fluids, the heat transfer coefficient obtained will be a consequence of a variety of related influences arising from minimum drop radius, surface inclination, interfacial properties, and degree of subcooling, apart from thermodynamic and thermophysical properties. Hence, it is preferable to cast these correlations in dimensionless form for the three liquid metals, applicable for all inclinations, saturation temperatures and degrees of subcooling. These expressions are

Mercury: 
\[ \text{Nu} = \left( 7.8 \times 10^3 \text{Ja} + 24.12 \right) \left( \frac{T_{\text{sat}}}{T_{\text{ref}}} \right)^{0.64} \left( \frac{r_{\text{crit}}}{r_{\text{cap}}} \right)^{-0.7} \]  
(21a)
FIG. 18: Variation in average heat transfer coefficient with saturation temperature underneath horizontal and over vertical substrates: (a) mercury, (b) potassium, and (c) sodium.

FIG. 19: Effect of degree of subcooling on the heat transfer coefficient at a saturation pressure of 50 kPa for three metals on a vertical substrate.

Potassium: \[ \text{Nu} = (3.12 \times 10^4 \text{Ja} + 58.71) \left( \frac{T_{\text{sat}}}{T_{\text{ref}}} \right)^{0.75} \left( \frac{r_{\text{crit}}}{r_{\text{cap}}} \right)^{-0.51} \] (21b)
Sodium: \( \text{Nu} = \left(3.28 \times 10^4 J_a + 61.92\right) \left(\frac{T_{\text{sat}}}{T_{\text{ref}}}\right)^{0.78} \left(\frac{r_{\text{crit}}}{r_{\text{cap}}}\right)^{-0.46} \) \hspace{1cm} (21c)

Eqs. (21a)–(21c) have correlation coefficients better than 98%. The reference values used for nondimensionalization at the normal boiling point of the liquid metals are given in Table 2.

### 6.1 Substrate Shear Stress

Figure 20 depicts the temporal variation in average shear stress on a vertical condensing wall, calculated from Eq. (15), during condensation of the three metal vapors at conditions corresponding to the data in Figs. 14–16, respectively. As shown, the average shear stress varies with time depending on the number of drops attaining criticality and moving

![Graphs of instantaneous surface-averaged wall shear stress variation on a vertical condensing surface: (a) mercury, (b) potassium, and (c) sodium.]

**FIG. 20:** Instantaneous surface-averaged wall shear stress variation on a vertical condensing surface: (a) mercury, (b) potassium, and (c) sodium.
on the substrate. The average shear stress on the substrate is a maximum in the case of mercury (48 mN/m$^2$); corresponding values for potassium and sodium are 8.75 mN/m$^2$ and 9.5 mN/m$^2$, respectively. Accordingly, the average skin friction coefficient, estimated from Eq. (16) is $1.97 \times 10^{-3}$ (mercury), $1.76 \times 10^{-3}$ (potassium), and $1.81 \times 10^{-3}$ (sodium).

7. CONCLUSIONS

A mathematical model of metal vapor condensation underneath horizontal and inclined surfaces is reported. The process starts at the nucleation of liquid droplets and proceeds to direct condensation of vapor, growth by coalescence, and fall-off/slide-off by gravity. At this point, a fresh cycle of condensation is initiated. Drop motion is independently simulated by a Navier-Stokes and energy equations solver. Wall shear stress and heat flux are included in the model as correlations. The drop size at criticality (either fall-off or slide off as the case may be) is determined from a force balance equation and is provided in the model as a correlation. The model predictions are in good agreement with the experiments reported in the literature for water and mercury. Using this approach, quantities of interest, such as drop size at criticality and average heat transfer rate, are predicted. The following conclusions can be drawn from the study:

- The condensation cycle times for liquid metal vapor are smaller when compared with that for water. Minimum drop radius, maximum drop size, and area coverage are also smaller. All of these factors result in a higher heat transfer coefficient.

- Among liquid metals, sodium has distinct properties: a very small drop radius at nucleation, a larger drop size at criticality, and a large cycle time. These factors indicate a low heat transfer coefficient. However, because all three metals have high thermal conductivity, they eventually arrive at comparable heat transfer coefficients.

- In contrast to sodium, mercury has a large drop radius at nucleation, a small drop radius at criticality, and a smaller cycle time. These factors indicate a high heat transfer coefficient. However, because mercury has a low thermal conductivity, its heat transfer coefficient is comparable with that of sodium.

- For an inclined surface, the condensation cycle is governed by drop instability, and the three metals yield similar condensation patterns and heat transfer rates.

- A high nucleation site density leads to a high overall heat transfer coefficient. Similarly, heat transfer rates increase with saturation temperature and degree of subcooling.

- The heat transfer coefficient of condensing metal vapor strongly depends on the orientation of the substrate. As a rule, an inclined surface has a higher heat transfer coefficient compared with that of a horizontal surface, becoming maximal for a vertically oriented surface.

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REFERENCES


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