INVESTIGATION OF THE VELOCITY FIELD IN THE WAVE RIVULET FLOWING DOWN A VERTICAL PLATE

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Complementary measurements of the thickness and velocity fields in wavy rivulets flowing down a vertical plate are conducted using particle image velocimetry and laser-induced fluorescence field techniques. Stagnant areas, in which the liquid flows with the velocity of the waves, are detected under the crests of large-amplitude, low-frequency waves. Longitudinal velocity component profiles are obtained within different parts of the waves. The most noticeable deviations of the velocity profiles from self-parabolic profiles are observed in the region of the capillary precursor and in stagnant areas under the wave crests.

KEY WORDS: rivulet flow, regular waves, laser-induced fluorescence, velocity field

1. INTRODUCTION

The fluid flow over a solid surface under the action of gravity in the form of a thin rivulet is called rill flow, or more often, rivulet flow. At moderate liquid flow rates, the rivulet flow always occurs when liquid flows over a hydrophobic surface and is often observed even on a perfectly wetted surface due to the destruction of the continuous film flow (Johnson et al., 1999).

Studying the rivulet flow over a flat inclined surface is a challenging task due to the instability of the most flow regimes, which is caused by wetting angle hysteresis (Kim et al., 2004). Even more challenging is the study of rivulets formed during the flow of a liquid film over structured surfaces used in packings for mass-transfer applications. Similar to the case of liquid films flowing down an incline (Alekseenko et al., 1994), waves can evolve on the free surface of rivulets, which can affect both the transport processes and the features of the flow regime. A wide range of flow regimes of wavy rivulets strongly depends on the flow nature in the area of the contact line, fluid physical properties, and wave excitation conditions (Young and Davis, 1987; Doniec, 1988).

Statistically reliable experimental data on the wave structure on the free surface of rivulets can be obtained under the conditions when the contact lines of the rivulet are straight. For example, such flow is formed at jet irrigation of an inclined cylinder, where the fluid collects in its lower part and moves in the form of a stationary rivulet with constant width (Kuibin, 1996). A model of wave motion on the free surface of such rivulets was developed by Geshev and Kuibin (1995). Wavy rivulets, flowing down along the bottom part of the inclined cylinder, were investigated experimentally by Alekseenko et al. (1996). The difficulties in experimentally studying the wave modes of rivulet flow over inclined surfaces is due to the three-dimensional (3D) shape of the developing waves and the small thickness of the liquid layer in wavy rivulets. A comprehensive description of these modes requires the use of field measurement techniques, such as particle image velocimetry (PIV) to measure the flow velocity field or the laser-induced fluorescence (LIF) technique to obtain the wave shapes. The customization of these methods in order to study rivulet flow is faced with intrinsic problems of methodological nature.
The PIV technique (Raffel et al., 1998), well proven in gas–liquid flow, has some serious constraints when used for film flow, and in particular for rivulet flow. These constraints are related to the need to account for optical distortion at the phase interface, the small thickness of the liquid layer, as well as high transverse velocity gradients in wave rivulets. In the literature, there is a lack of data in relation to the experimental investigation of the velocity field at rivulet flow. This is because of the complexity of compensating for optical distortions associated with the 3D structure of the waves on the rivulet free surface. The only study carried out in this area is that by Alekseenko et al. (2007), in which a method for measuring the velocity field in the wavy rivulet flowing down along the lower part of the inclined cylinder was proposed.

The LIF technique is based on the reconstruction of the film thickness through intense radiation emitted by a small amount of fluorescent dye, which is dissolved in a working fluid. This method allows a field of local liquid film thicknesses over a large area to be recorded. The first field measurements using the fluorescence method were performed in the study of waves on liquid films flowing down an inclined plate (Liu et al., 1993). With respect to the rivulet flow, this method was used by Johnson et al. (1999) to study the formation of rivulets in films flowing down an inclined plate and constrained by a contact line. Field measurements of the wave shape on the rivulets flowing down a vertical plate were originally performed by Alekseenko et al. (2010) using the LIF technique. This paper presents the results of measurements of the velocity field in wavy rivulets with straight contact lines flowing down a vertical plate using the PIV and LIF field methods, which complement each other.

2. SCHEME OF INSTALLATION AND EXPERIMENTAL PARAMETERS

The rivulet flow was organized on a special-purpose installation with a closed hydrodynamic loop and replaceable test sections made of 4 mm-thick plate glass. We investigated the rivulet flow bounded by straight parallel waterfronts made out of hydrophobic material. Fluoroplastic film (10 µm thick), rolling down the surface of the test section, was used as the hydrophobic material. At different parts of the test section, the distance between the waterfronts varied within the range of 3–15 mm, with 4 mm increments. A general diagram of the experimental setup is shown in Fig. 1. Regular waves with predetermined frequency $F$ were created by modulating the liquid flow rate at the inlet of the distribution section.

3. OPTICAL ARRANGEMENT OF THE EXPERIMENT AND MEASURING SYSTEM

When measuring the velocity fields in the rivulets using the PIV method we faced the problem of determining the position of the rivulet free surface as well as the phase velocity of the waves. This problem was solved with the

**NOMENCLATURE**

- $b$: rivulet width (mm)
- $C$: phase velocity of waves (m/s)
- $F$: frequency of waves (Hz)
- $g$: gravitational acceleration (m/s$^2$)
- $H$: maximal value of $h$ (mm)
- $h$: film thickness in the central cross section of the rivulet (mm)
- $k$: image compression ratio
- $Q$: liquid flow rate (mL/s)
- $Re$: Reynolds number $[= g H^3 / (3 \nu^2)]$
- $U$: longitudinal velocity component (m/s)
- $U_0$: maximal value of $U$ (m/s)
- $x$: longitudinal coordinate (mm)
- $y$: normal to the plate coordinate (mm)
- $z$: transversal coordinate (mm)

**Greek Symbols**

- $\alpha$: rivulet contact angle ($^\circ$)
- $\beta$: observation angle for the laser-induced fluorescence technique ($^\circ$)
- $\gamma$: observation angle for the particle image velocimetry technique ($^\circ$)
- $\nu$: kinematic viscosity (m$^2$/s)
- $\rho$: density (kg/m$^3$)
- $\sigma$: surface tension (kg/s$^2$)
simultaneous application of the PIV and LIF techniques because the LIF technique allows determining with high accuracy the shapes of the waves, and accordingly their phases and velocities. In order to exclude optical distortions arising due to the undulated free surface of the wavy rivulet, the measurements were carried out from the non-irrigated side of the test section. An optical diagram of the experiment is shown in Fig. 2.

The main elements shown in Fig. 2 are described as follows: On the transparent plate (No. 1) a wavy rivulet (No. 2) bounded by 10 µm-thick hydrophobic waterfronts (No. 3) flows downward. The laser (No. 4) is equipped with a cylindrical lens to form a light sheet (No. 5) with a width of 0.8 mm and opening angle of 30°. The light sheet illuminates the rivulet along the flow direction in the central section, where the liquid layer thickness is maximal. Recording of the LIF images is carried out with the charge-coupled device (CCD) camera (No. 6). Observation angle \( \beta \) of the LIF-related camera is chosen as the smallest possible for a particular system layout. A more detailed description of the LIF technique with regard to the study of rivulet flow is given in Alekseenko et al. (2010). The CCD camera (No. 7) is used to capture the tracer field with subsequent determination of the liquid velocity field using the PIV technique. Both cameras are equipped with orange filters (No. 8), which suppress the reflected and scattered laser radiation.

To implement the measurements using the PIV and LIF techniques, Rhodamine 6G fluorescent dye was dissolved in a working fluid in the amount of 0.05 mg/L. Polyamide particles with a diameter of 5–15 µm, colored by Rhodamine 6G, and with a density close to the density of the working fluid, were used as the tracers in the PIV technique. Aqueous glycerol solution with a kinematic viscosity of \( \nu = 3.92 \times 10^{-6} \) m²/s, density of \( \rho = 1130 \) kg/m³, and surface tension of \( \sigma = 0.07 \) kg/s² was used as the working fluid.

In this work, we used a Polis measuring system equipped with a double Nd:YAG laser with a pulse energy of 50 mJ and wavelength of 532 nm; a control processor and data processing system (Polis-3); two special purpose digital CCD cameras (Kodak Megaplus ES 1.0), which allowed operation in the dual frame mode with a resolution of 1008 × 1018 pixels; as well as the ActualFlow 1.17.2.11 software package. The software was used to monitor the control system.
when measuring and calculating the velocity field using the PIV technique, as well as the filtering and interpolation. Calculation of the instantaneous velocity fields was conducted using a standard cross-correlation algorithm with a resolution of $8 \times 64$ pixels and 50% overlap (Raffel et al., 1998). Measurements of the thickness field in the dual frame mode with a known time delay made it possible to determine both the shape and wave phase velocity $C$.

To achieve sufficient resolution of the velocity field over the rivulet depth, a PIV-related camera should be positioned at a larger angle $\gamma$; in which the image of the observed area is compressed in the transverse direction. Image compression ratio $k$ depends on $\gamma$ and the refractive indices of the liquid and glass and can be calculated based on geometrical optics principles. However, under experimental conditions, it is more convenient to define the compression ratio by direct measurement. For this purpose, a glass plate with a micrometer grid was submerged into the working fluid and fixed in the rivulet cross-sectional plane, corresponding to laser sheet. The glass plate was photographed at several values of angle $\gamma$. As a result, $\gamma = 35^\circ$, at which $k = 2.92$, was chosen for the experiments. Recording with the LIF-related camera was carried out at a spatial resolution of 84.3 $\mu m/pixels$, while by recording with the PIV-related camera was carried out at a spatial resolution of 5.3 $\mu m/pixels$.

Successive stages of determining the free boundary of the wavy rivulet in its central section using the LIF technique is shown in Fig. 3. Instantaneous wave rivulet profiles obtained by the LIF technique were used to mask the images of

![FIG. 2: Optical diagram of the experiment: 1, transparent vertical plate; 2, rivulet; 3, waterfaces from the fluoro-plastic film; 4, laser; 5, laser sheet; 6, CCD camera for recording the thickness field using the LIF technique; 7, CCD camera for capturing marker field using the PIV technique; 8, orange light filters; $\alpha$, rivulet contact angle; $\beta$, observation angle for the diagnosis using the LIF technique; $\gamma$, observation angle for the diagnosis using the PIV technique.](image)

![FIG. 3: Determination of the wave rivulet boundary using the LIF technique: (a) primary LIF image; (b) reconstructed free surface of the rivulet in the vicinity of its central section; (c) longitudinal profile of the wave in the central section of the rivulet.](image)
the PIV-related camera. After that, the luminous background formed by the fluorescent glow of the working fluid was removed and the image contrast was enhanced. The prearrangement of images obtained by the PIV-related camera for the subsequent calculation of the velocity field is shown in Fig. 4.

To obtain statistics on the velocity field in a certain phase we used the phase-averaging method. The sampling frequency of the PIV system was set at a level that was different from the wave excitation frequency on the rivulet surface, \( F \), using the value of \( 0.05F \). The obtained images of the tracer fields were distributed into image groups with close wave phases. A liquid velocity field was constructed for each group. A statistically averaged velocity field was calculated based on 30–50 instantaneous realizations. Finally, adjustments to the average velocity field were performed in accordance with a compression ratio of \( k = 2.92 \), which was obtained during the calibration of the optical system.

4. MEASUREMENT RESULTS

As was shown by Alekseenko et al. (2010), wave pattern peculiarities in the case of rivulet flow compared with film flow are observed at low wave excitation frequencies when large-amplitude waves develop on the rivulet surface. The shape of these waves is quite similar to the step with a steep leading edge and gently sloping wave tail. For such waves one should expect a strong deviation of the liquid flow profile from a parabolic shape, which is often used in modeling film flows (Alekseenko et al., 1994). For this reason, much attention was directed to the study of velocity fields in wavy rivulets at low wave excitation frequencies.

Step-like waves were observed in the rivulets with different widths within a rather wide range of liquid flow rate \( Q \) at excitation frequencies \( F \) within the range from 0.5–1.0 to 25–30 Hz. Higher frequencies led to the development of waves with small amplitudes, in which the shape was close to a sinus-like wave. In the absence of external excitation, stable flow of smooth (waveless) rivulets was observed. In this case the cross section of the rivulet is an arc of a circle, and the wetting angle, \( \alpha \), is determined from the expression \( \tan \alpha = 2hb/(b^2 - h^2) \), where \( b \) is the rivulet half-width and \( h \) is the waveless rivulet thickness along its centerline, measured using the LIF technique. The Reynolds number of the rivulet flow was defined as \( \text{Re} = gh^3/(3\nu^2) \).

Our measurements have shown that the velocity fields of liquid flow at low wave excitation frequencies have almost the same structure at different liquid flow rates and rivulet widths. This structure markedly differs from that observed in the case of waves developed on the surface of a rivulet flowing down along the lower part of the in-

![FIG. 4: Prearrangement for recording using the PIV-related camera: (a) original image; (b) masking of an image using the wave profile obtained by the LIF technique; (c) background removal and image contrast enhancing.](image-url)
clined cylinder, where pronounced swirling motion was observed in the vicinity of wave crest (Alekseenko et al., 2007).

The characteristic pattern of the velocity field in the vicinity of the leading edge of the low-frequency wave at different liquid flow rates and rivulet width of 7 mm is shown in Figs. 5–7. The wave maximum corresponds to \( x = 0 \). As is obvious from Figs. 6 and 7, in a reference frame moving with the wave velocity there is a stagnant region with weak stochastic fluid motion under the wave crest. A similar pattern was observed at low excitation frequencies in all of the investigated rivulet flow regimes within the range \( 40 < \text{Re} < 200 \). Extension of the stagnant region had no pronounced dependence on \( \text{Re} \) and \( F \) and was equal to 2.5–3.5 mm.

The liquid velocity profiles that are typical for various low-frequency waves are shown in Figs. 8 and 9, where the solid lines show self-similar parabolic profiles for the respective sections constructed based on maximum rivulet thickness \( H \) and measured liquid superficial velocity \( U_0 \):

\[
U = 2U_0 \left[ \frac{y}{H} - \frac{1}{2} \left( \frac{y}{H} \right)^2 \right]
\]  

(1)

**FIG. 5:** Velocity field in the vicinity of the leading edge of the wave: \( Q = 1.8 \text{ mL/s}; \text{Re} = 97; \alpha = 12.6^\circ; F = 12 \text{ Hz}; C = 0.8 \text{ m/s}.\)

**FIG. 6:** Velocity field in the vicinity of the wave leading edge in the reference frame moving with the wave velocity: \( Q = 1.8 \text{ mL/s}; \text{Re} = 97; \alpha = 12.6^\circ; F = 12 \text{ Hz}; C = 0.8 \text{ m/s}.\)
FIG. 7: Velocity field in the vicinity of the leading edge of a wave in the reference frame moving with the wave velocity: $Q = 0.8 \text{ mL/s}; \text{Re} = 46; \alpha = 9.8^\circ; F = 4 \text{ Hz}; C = 0.73 \text{ m/s}.$

FIG. 8: Longitudinal velocity profiles in the sections intersecting the wave at its maximum (section 1) and at the capillary precursor minimum (section 2); the maximal total measurement error was estimated to be equal to 10% relative to the maximum longitudinal velocity (Alekseenko et al., 2007): (a) $Q = 1.8 \text{ mL/s}; \text{Re} = 97; \alpha = 12.6^\circ; F = 12 \text{ Hz}; C = 0.8 \text{ m/s}$ (coordinate of precursor minimum $x = 2.75 \text{ mm}$); (b) $Q = 0.8 \text{ mL/s}; \text{Re} = 46; F = 4 \text{ Hz}; \alpha = 9.8^\circ; F = 0.73 \text{ m/s}$ (coordinate of precursor minimum $x = 2 \text{ mm}$).

The dashed lines show the profiles obtained from the Nusselt formula for laminar film flow:

$$U = \frac{gH^2}{\nu} \left[ \frac{y}{H} - \frac{1}{2} \left( \frac{y}{H} \right)^2 \right] \quad (2)$$

As is obvious from Fig. 8, the largest deviations from the parabolic shape in the measured velocity profiles are observed in the vicinity of capillary precursor and in the stagnant area under the wave crest. The typical velocity profiles for the wave tail and the residual liquid layer are shown in Fig. 9. At distances of 2–3 mm behind the wave maximum, the velocity profile is well described by the self-similar profile [Eq. (1)], whereas at distances of 5–6 mm it is close to the profile described by the Nusselt formula [Eq. (2)].
5. CONCLUSIONS

With the use of the PIV and LIF field techniques, we have carried out complementary measurements of thickness fields and liquid flow velocity fields in wavy rivulets flowing down a vertical plate. At low wave excitation frequencies, we observed a stagnation region under the wave crests, where the fluid moves with the wave velocity. The length of the stagnation region does not have a pronounced dependence on the flow operating parameters and is 2.5–3.5 mm. Longitudinal velocity component profiles were obtained for liquid flow in different parts of the waves. The largest deviations of the velocity profiles from the self-parabolic profiles were observed in the area of the capillary precursor and in the stagnant regions under the wave crests.

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