PROGNOSIS AND NUMERICAL ANALYSIS OF THE PRESSURE DROP IN SLUSH HYDROGEN VACUUM-JACKETED TRANSFER LINES

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Hydrogen offers high combustion qualities and therefore is chosen as a propellant for the national aerospace plane (NASP). The low density of hydrogen is a major inconvenience; therefore, a solid–liquid mixture called SLUSH hydrogen is used to increase the density and cooling capacity. In this paper, a two-phase mixture is modeled using separated flow model in which the mathematical equations are written separately for each phase where different properties and velocities are considered for each phase. Mass, momentum, energy equations, and interfacial phenomena equations are developed with the inclusion of drag force, virtual mass force, mass and momentum transfer, and interfacial shear stress. Turbulence effects are treated and multiparticle drag correlations are used. Results for pressure drop across a 1.5 in. Schedule 5S vacuum-jacketed pipe show good agreement comparing it with earlier experimental data and numerical analysis.

KEY WORDS: SLUSH hydrogen, solid fraction, flow characteristics, multiphase flow, CFD analysis

1. INTRODUCTION

Numerical simulation of slush hydrogen flow field has not been the subject of many numerical and 3D modeling studies in the literature. In the sixties, experimental studies have been performed as part of the National Aerospace Plane (NASP) program to freeze hydrogen properties. The first studies reporting on numerical activities for slush were published in the late eighties, and the first numerical examination for slush hydrogen production and utilization had been elaborated and published only in 1994. Thus, the studies found in the literature reporting some efforts predict slush flows using computational fluid dynamics (CFD) approaches have been carried out in the frame of two-dimensional (2D) analysis by applying a single-phase flow. Few efforts have been conducted to simulate the slush flows by applying the two-phase flow. In fact, two-phase flows similar to single-phase flows obey the basic laws of fluid mechanics. However, the equations that characterize a two-phase flow are more numerous and more complicated. Techniques range from simple to more sophisticated based on the amount of variables needed to define the two-phase flow. These techniques can be divided into multiple categories as follows: correlations, simple analytical models, integral analysis, differential analysis, and universal phenomena. In this paper, the analysis is based on the simple analytical models. These models are divided into three methods called homogeneous model, drift-flux model, and separated-flow model. The homogeneous model provides the simplest analytical technique to describe a two-phase flow as the mixture is assumed to behave as a pseudo-fluid that follows the single-phase flow equations. In this model, the properties needed are velocity, thermodynamic properties, and transport properties. The drift-flux model focuses on the relative motion between the two phases. Furthermore, the separated-flow model assumes the phases to flow side by side. Therefore, separate equations are used for each of the phases, and each phase has different properties and different velocities. This technique leads to six equations that are going to be solved simultaneously along with rate equations which describe how the phases interact with each other and with the walls of the duct (Wallis, 1969). This technique is going to be used throughout this paper in addition to interfacial equations.
Sindt and Ludtke (1969) studied the characteristics of the solid–liquid mixture, preparation techniques, transfer and pumping, aging effects, and instrumentation needed. The study discussed the preparation of slush hydrogen by periodic vacuum pumping, also known as the freeze–thaw method. This preparation method was developed by Mann et al. (1969) to reduce hydrogen partial pressure below the triple-point pressure during pumping. Permissible aging effects in this mixture were achieved, and it has been observed that the slush was aged as much as 100 h during this period. Moreover, the solid particle was detected to a significant change where it becomes more spherical in shape with a value of fresh mixture solid fraction between 0.35 and 0.45 and with an optimum value of 0.6 completed in 50 h. Sindt and Ludtke (1973) considered the settling velocity of the slush particle. They found out that the settling velocity depends on the particle size and follows the relationship of turbulent settling. As for the transfer and pumping characteristics, Sindt confirmed that flow losses in 10 h aged mixture are 4 to 10% higher than the losses in 1 h old slush. Pressure losses during transfer were measured in a pipe for triple-point liquid and a 0.55 slush mixture. The study concluded that if the Reynolds number is kept high into the turbulent region, the losses of slush will be approximately equal to the losses in triple-point liquid hydrogen. Furthermore, Sindt and Ludtke (1969) performed a study on solid fraction upgrading in which a solid-fraction-upgrading Dewar was added to an existing test facility.
The results showed that upgraded aged slush hydrogen attained a 0.63 solid fraction using the Dewar, and upgraded fresh mixture with a solid fraction of 0.53 retains enough fluidity to be mixed and transferred through lines, valves, and other flow restrictions. Ewart and Dergance (1978) published a study on flow calculation for the use of cryogenic propellants on advanced space transportation systems. However, this study was determined on flows in pipe sizes above 6 inches. Hardy (1990) derived the first engineering approach to calculate the pressure drop and solid fraction loss for a one-dimensional steady-state slush hydrogen flow through pipe systems. In his study, a code was compiled named FLUSH (flow of slush hydrogen) that solves the Bernoulli and the energy equation in each element by resolving equations at two nodes located at the inlet and the exit of every single element. The analysis also calculates the heat leak across vacuum-jacketed or normal insulation pipes where it is assumed that the primary source for heat leak across vacuum-insulated pipes is due to radiation and adopts that all the heat is used to melt the solid. Therefore, the temperature is taken constant at any section of the element. Further, FLUSH assumes that the mixture viscosity is equal to that of liquid hydrogen. At low flow rates, FLUSH results show good agreement with experimental data published by National Aeronautics and Space Administration (NASA). However, at high flow rates, results deviate dramatically from experimental data and this is due to the turbulence effects in which the solid particles come to be overflowing. This deviation in results is clarified because FLUSH does not assume a separated two-phase flow model in which the velocity is assumed the same for the two phases. In addition, FLUSH does not take into account turbulence nor interfacial phenomena such as heat, mass transfer, and shear stress. Also, a one-dimensional model does not predict good results in large pipes due to the velocity gradient in the normal direction to the pressure gradient, wall friction, and turbulence. Hardy (1990) concluded that at low flow rates, the effects of heat transfer dominate the loss in solid fraction. At high flow rates, friction heating dominates the solid fraction loss. Moreover, when the pipe diameter increases, the solid hydrogen degradation decreases and this is due to the decrease in friction losses. Navickas et al. (1989) conducted numerical studies to estimate the liquid thermal stratification caused by buoyancy effects. This approach is based on a two-phase flow analysis in which one set of equations is provided for each phase. Furthermore, a drag coefficient and an effective mixture viscosity were introduced based on a multiparticle drag correlation provided by Ishii and Chawla (1979). The mixture viscosity was estimated by the following power law:

$$\mu_m = \mu_l \left(1 - \frac{\alpha_s}{\alpha_{sm}}\right)^{-2.5\alpha_m}$$  \hspace{1cm} (1)

where $\mu_m$ is the mixture viscosity, $\mu_l$ is the liquid viscosity, $\alpha_s$ is the solid fraction, and $\alpha_{sm}$ is the maximum solid packing fraction, which is given by Ishii and Kataoka (1983) as 0.62 for spherical particles. Zakhia (2001a) developed the first numerical simulation of slush hydrogen flow in pipes available in the literature. Zakhia (2001c) used PHOENICS, a general CFD program, as a tool to model the flow of the solid–liquid mixture in vacuum-jacketed piping systems of 1.5 inch to 4 inches. Multiparticle drag correlation was used along with the separated two-phase flow model in addition to the interfacial and turbulence effects. Zakhia (2001b) used a 2D separated two-phase flow model in which equations for mass, energy, and momentum conservation are used for each phase. In addition, interfacial pressure, mass transfer, and shear stress were added to the analysis. The equations were solved using the Eulerian–Eulerian technique, which uses a fixed grid and employs the concept of “interpenetrating continua.” The phase volume fractions equation in steady-state form follows the relation

$$\nabla (\alpha_i \rho_i V_i) - \nabla (G_{\alpha i} \nabla \alpha_i) = m_{ji}$$  \hspace{1cm} (2)

The first term represents the convection term, the second one represents the phase diffusion, and the term on the right-hand side represents the net rate of mass transfer. Here $\rho_i$ is the phase density, $V_i$ is the phase velocity vector, $G_{\alpha i}$ is the diffusion coefficient, and $m_{ji}$ is the net rate of mass entering phase from other phase. Zakhia (2001a) assumes that the solid pressure at the interface is equal to the bulk solid pressure ($P_{s,i} = P_s$). Also, the model assumes that the solid pressure at the interface is equal to the liquid pressure at the interface where

$$P_l = P_{l,i} + \xi_l \rho_l |V_r|^2$$  \hspace{1cm} (3)

where $P_l$ is the bulk liquid pressure, $P_{l,i}$ is the liquid pressure at the interface, $V_r$ is the relative velocity, and $\xi_l$ is a coefficient given by Stuhmiller (1977) for flow over a sphere and is equal to 1/4. Also, Zakhia (2001a) used the
interfacial drag force models adopted by Ishii and Chawla (1979) and the time-averaged Navier–Stokes equation where the models adopt the $k$-$\varepsilon$ model.

In Japan, Ishimoto and Ono (2005) have developed some studies on slush nitrogen flows in pipes. These studies were funded by Japan Aerospace eXploration Agency (JAXA). Even though the studies predicted slush nitrogen flow in pipes, they are very important due to the detailed numerical model included. The 2D mass conservation, motion, and energy equations are written based on the Eulerian–Lagrangian technique with unsteady thermal non-equilibrium. The study took into account the solid fraction, duct shape, and the thermodynamic behavior of slush nitrogen. The model assumes that the solid particles are spherical in shape and takes into account the energy transfer at the interface; however, phenomena such as liquid vaporization are not considered. Gravitational forces are included by applying a two-way coupling technique and hence replacing the motion equation by the translational motion of a single rigid particle. The terms of this equation are nearly similar to the momentum equation terms found in previous studies (Zakhia, 2001a). However, the solid particle diameter was evaluated using an empirical formula given by Tao (2003):

$$d_p = \left(1 - \frac{\Gamma_p \Delta t}{\beta \alpha_p \rho_p}\right)^{1/3}$$  \hspace{1cm} (4)

where $\Gamma_p$ is the phase generation density and $\Delta t$ is the time increment.

Simulations were performed on a straight pipe and on a converging–diverging duct. The results on the converging–diverging pipe showed that the solid particles are concentrated in the vicinity of the lower wall because gravitational forces are more important. By using a straight pipe, results showed that the pressure loss is possibly decreased by decreasing the apparent viscosity. The most important numerical results also proved that the pressure loss in slush nitrogen is lower than that of a single-phase liquid nitrogen, and it is found that the pressure loss is decreased when the pipe length is increased. Those numerical results are compared with experimental results where it was found that for high Reynolds numbers the pressure loss is reduced by using a two-phase slush flow. Recently, Ohira (2004) worked on the measurement of the density and the mass flow rate of slush hydrogen. The study discussed densimeters types, the capacitance type, and the microwave type. Further, it discussed the development of the Slush hydrogen flowmeter stating also the capacitance and the microwave type. Experiments were conducted to conclude that measurement accuracy of about 0.5% was achieved by using a capacitance or a microwave-type densimeter. A three-dimensional (3D) numerical study on cryogenic fluids such as slush hydrogen and slush nitrogen was conducted by Ohira et al. (2012) in horizontal circular pipes. The authors achieved a synergetic effect by combining the slush hydrogen and high-temperature super-conducting equipment. A numerical code was developed called “SLUSH-3D” in which basic equations for a 3D two-fluid analytical model are implemented. However, this numerical model did not take into account some phenomena and treated the fluid as Newtonian. The effective kinematic viscosity was used to take into account turbulence effects.

$$v_{t}^{eff} = v_t + v_t^l = v_t + C_{\mu} \frac{k^2}{\varepsilon}$$  \hspace{1cm} (5)

$$v_s^{eff} = v_s + C_t v_t^l$$  \hspace{1cm} (6)

where $C_{\mu} = 0.09$, $C_t = 1.0$, $v_s = 10. v_t$.

The finite volume method was used to discretize the variables. The OpenFOAM is used as the coding for the basic CFD solver; calculations were made by varying the inlet velocity for two diameters of slush nitrogen particles depending on the method of production. However, changes in the thermo-physical properties for both phases due to the change in temperature during heating were not considered. Experiments were conducted on a 15 mm diameter horizontal pipe 600 mm in length and the results are compared with the numerical results. Results showed that the flow acts as pseudo-homogeneous flow at high flow rates and approaches that of a single-phase liquid nitrogen. In addition, at high flow rates, particle size has nearly no effect on the velocity distribution, whereas at low flow rates, the solid particles are concentrated in the bottom because gravitational forces are larger than inertial ones. This leads to a higher loss in momentum and an increase in drag forces. Also, at these flow rates results showed that when the particle diameter is small the flow is more likely to approach symmetry. Slush hydrogen and slush nitrogen are proven to be very similar in behavior while the solid fraction of slush hydrogen is more uniform than that of slush nitrogen at low flow rates. In addition, because the viscosity of liquid hydrogen is about 11 times lower than that of liquid hydrogen.
nitrogen, the drag between the phases is reduced and hence, the velocity distribution of solid hydrogen can reach a 
pseudo-homogeneous state easier than that of nitrogen. As for the pressure drop, results showed that the pressure drop 
increases with inlet velocity and solid volume fraction for slush hydrogen and slush nitrogen. The rate of increase in 
pressure drop for slush hydrogen and slush nitrogen is defined by

$$\Delta P_{\text{increase}} = \left( \frac{\Delta P_{sl}}{\Delta P_l} - 1 \right) \times 100$$  \hspace{1cm} (7)$$

where $\Delta P_l$ is the pressure drop of the liquid phase and $\Delta P_{sl}$ is the pressure difference between the phases. As the 
solid fraction at the inlet increases, the turbulent energy and the pressure drop increase.

In Europe, many studies on slush flows in pipes were done under the patronage of the Future European Space 
Transportation Investigations Programme (FESTIP) at the University of Rome (Casali and Gamma, 2000; Crivellari 
et al., 1999; Gamma and Del Monte, 1998). These studies used the STAR-CD tool and the governing equations are 
based on the Lagrangian–Eulerian model. The model takes into account turbulent, mass, momentum, and energy 
interactions between the phases. STAR-CD uses a statistical approach under the computational parcel concept, where 
the analysis is done on a sample of the entire particles population. The turbulent interaction between the phases is 
modeled by the following equation:

$$m_d \frac{du_d}{dt} = F_{dr} + F_p + F_{am} + F_b$$  \hspace{1cm} (8)$$

where $m_d$ is the computational parcel of mass, $u_d$ is the instantaneous velocity of the fluid and of the particle, $F_{dr}$, 
$F_p$, $F_{am}$ and $F_b$ are the drag, pressure, virtual mass, and the body force, respectively. This interaction between phases 
takes place over a time span equivalent to the minimum of the two representative time intervals. Calculations were 
done on triple-point hydrogen; the flow is considered as steady, viscous, and incompressible. Gravitational forces 
were taken into account and the $k-\varepsilon$ model was used for turbulence. The study used experimental data from NASA to 
validate some empirical correlations that were used to determine losses in a straight line. A correlation was used to 
determine the drag coefficient when the Reynolds number increases, because inertial effects dominates viscous forces.

$$C_d = \left( \frac{4.8}{Re^{1/2} + 0.632} \right)^2$$  \hspace{1cm} (9)$$

This correlation covers Reynolds number ranges up to $2 \times 10^5$. The apparent viscosity is expressed as a function of 
solid concentration and is given by the following equation:

$$\mu_{\text{conc}} = \mu_l e^{(5F_s)/(3(1-F_s))}$$  \hspace{1cm} (10)$$

where $\mu_l$ is the liquid viscosity and $F_s$ is the solid concentration. Results showed that when the solid volume fraction 
is close to 0.5, the slush behavior is nearly similar to that of liquid hydrogen at triple point. The results of the simulation 
were compared with pressure drops calculated from the correlations. The pressure losses were found practically 
similar above the critical speed. Recently in Europe, Reynier and Bugel (2011) have done an important review on the 
modeling of slush in addition to an engineering numerical model in the frame of ESA Future Launcher Preparatory 
Programme (FLPP). At first, the review described two-phase flow regimes in details where homogeneous, heterogeneous, 
saltation, and stationary bed were detailed along with their velocity profiles and their head losses. Second, a 
detailed literature review was conducted to start with the production of slush to old and recent studies done on slush 
hydrogen and slush nitrogen. Also, important works done by NASP, JAXA, and FESTIP programs were summarized. 
The study then described the FLUSH (Hardy, 1990) model and showed the available database. The database is a large 
set of experimental results derived from multiple experiences done on the production, instrumentation, aging effects, 
numerical results, and transfer methods of slush. A section is added to discuss how to validate the results, where two 
sets of data were used to achieve this task. Finally, two advanced numerical models were written, the first consists 
of the Eulerian–Lagrangian model and the second of the Eulerian–Eulerian model. It has been mentioned that the 
Eulerian–Lagrangian approach is better at low volume fractions and the Eulerian–Eulerian approach is better at high 
volume fractions. The same assumptions from Ishimoto and Ono (2005) are used where the particles are considered
spherical, interfacial effects are taken into account, and the liquid vaporization is neglected. Mass, combined equation of motion for the two phases and energy equation were written. The equation of motion for the slush particle is modified by using a two-way coupling technique to consider the effect of additional forces acting on slush particles. However, in this model, collisions between particles were not taken into account. In the Eulerian–Eulerian model, the dispersed flow equations are written separately for each phase, resulting in two equations of mass, momentum, and energy balance in addition to some interfacial, phase, or closure and rate equations. Also, gravitational forces, interfacial phenomena, and fluid–wall interactions were considered. Jin et al. (2017) worked on the numerical prediction of slush flow characteristics in a horizontal pipe. A literature review was written, briefly describing earlier numerical and empirical studies. This study was based on the Eulerian–Eulerian model for separate phases and yet intractable. The novelty of the model is that large-size solid hydrogen particles were considered in the calculation. An effective mixture viscosity is introduced to take the effect of particles on the interfacial momentum into consideration; also, the wall boundary conditions for the solid phase are based on the Johnson–Jackson model, which involves the collision between the particles and the wall. Also, particle–particle and particle–wall restitution coefficients are added. The variation in the mass properties due to temperature and evaporation are not taken into consideration. The effective mixture viscosity is derived from Einstein’s formula and used in the interfacial drag force equation:

\[
\mu_m = \mu_l e^{(2.5/\beta)(1\left(1-\alpha_s\right)^\beta-1)}
\]

where \(\beta\) is the only parameter that accounts for the particle shape and size; its value ranges from 0.95 to 3.9. The simulation is ran on a horizontal straight line 45 mm in diameter and 3 m in length.

The study gave results for solid fraction loss at the end section of the pipe, solid fraction concentration, pressure drop along the pipe, and velocity profiles. The validation of these results is with respect to earlier experimental data available in the literature; the model was capable of giving good results when the velocity is not too high. The pressure drop was generally found to be increasing when the inlet velocity is increased and when the solid particle size increases. Further, the particles tend to be more concentrated on the lower parts of the pipe due to the higher density of the solid phase; at high concentration, the velocity profiles become more symmetrical and on high velocities, the solid particles were found to be accelerating in the lower parts of the pipe leading to a pseudo-homogeneous flow. Finally, the pressure drop for slush hydrogen was found to be lower than that of subcooled liquid hydrogen. The study mentioned at last that further studies should be done to include heat transfer effects, pipe geometry, and geometric structure parameters.

2. MATHEMATICAL MODEL

2.1 Mass Conservation

The general form of the conservation of mass equations or continuity equations in the differential form:

\[
\frac{\partial}{\partial t} [\rho_1 (1 - \alpha)] + \nabla [\rho_1 (1 - \alpha) V_1] = S_{12} + S_1
\]

\[
\frac{\partial}{\partial t} (\rho_1 \alpha) + \nabla [\rho_2 \alpha V_2] = S_{21} + S_2
\]

where phase one and two are represented, respectively, by subscripts 1 and 2, \(\alpha\) is the fraction of an element of volume occupied by component number two, the terms \(S_{12}\) and \(S_{21}\) represent the mass rate of phase change at the interphase and \(S_{12} = -S_{21}\), \(S_1\) and \(S_2\) are external sources of matter and approximately equal to zero, \(V\) is the velocity for each phase. Hence, for a steady-state solid–liquid mixture, these two equations reduce to

\[
\nabla [\rho_1 \alpha_l V_l] = m_{ls}
\]

\[
\nabla [\rho_s \alpha_s V_s] = m_{sl}
\]

where \(\alpha_l\) and \(\alpha_s\) are the volumetric concentrations of the liquid and the solid phase, respectively. The mass transfer rate from a phase is assumed to be equal to the rate of mass transfer received by the second phase; in other words, the

Interfacial Phenomena and Heat Transfer
solid disappearance is equal to the liquid appearance. Therefore, \( m_{ls} = -m_{sl} = \Gamma \). The mass transfer depends on the area of the slush sphere, interfacial heat transfer coefficient, bulk and interfacial temperature, and the latent heat of fusion:

\[
\Gamma = m_{ls} = -m_{sl} = \frac{Q_{il} + Q_{is}}{H_{if}}
\]

where the heat transfer at the interface, \( Q_i \), is the sum of the heat transfer from the interface to the liquid phase and the heat transfer from the interface to the solid phase as

\[
Q_i = Q_{il} + Q_{is}
\]

The two components, representing the heat transfer between the interface and a phase, can be evaluated by a simple heat transfer equation as

\[
Q_{il} = \frac{h_{il}A_{il}}{C_{pl}V_{cell}} (H_l - H_{l, sat})
\]

\[
Q_{is} = \frac{h_{is}A_{is}}{C_{ps}V_{cell}} (H_s - H_{s, sat})
\]

where \( h_i \) is the interfacial heat transfer coefficient, \( H \) is the specific enthalpy of a respective phase, \( H_{sat} \) is the specific saturation enthalpy of a phase, \( C_p \) is the specific heat at constant pressure, \( A_{i} \) is the interfacial area, and \( V_{cell} \) is the volume of a cell. In order to evaluate the above equations, the interfacial heat transfer coefficients must be estimated by empirical expressions. At first, the interfacial heat transfer coefficient can be evaluated by

\[
h_{il} = \frac{Nu.k_l}{D_p}
\]

\[
h_{is} = \frac{Nu.k_s}{D_p}
\]

In Eqs. (19) and (20), \( Nu \) is the Nusselt number, \( k \) is the thermal conductivity, and \( D_p \) is the diameter of a particle. The empirical equations are used to evaluate Nusselt number. Zakhia (2001a) used equations from the literature and wrote for:

Laminar flow (\( Re_D < 23 \))

\[
Nu = 3.658 + \frac{0.0668 \,(d/L) \, Re_D \, Pr}{1 + 0.04 \, [(d/L) \, Re_D \, Pr]^{2/3}}
\]

Turbulent flow (\( Re_D > 2300 \))

\[
Nu = 0.023 \, Re_D^{4/5} \, Pr^{1/3}
\]

where Prandtl and Reynolds number are defined as

\[
Re_D = \frac{\rho_l V_l D_p}{\mu_l}
\]

\[
Pr = \frac{\mu_l C_{pl}}{k_l}
\]

Basic equations of phase closure, mixture density, thermal conductivity, and mixture velocity are given by

\[
\alpha_s + \alpha_l = 1
\]

\[
\rho_m = \alpha_s \rho_s + \alpha_l \rho_l
\]

\[
k_m = \alpha_s k_s + \alpha_l k_l
\]

\[
V_m = \alpha_s V_s + \alpha_l V_l = u_s + u_l
\]
2.2 Momentum Equation

The momentum equation of a two-phase flow in a general and 3D form is given by Ishii and Chawla (1979):

\[
\frac{\partial \alpha_k \rho_k \vec{V}_k}{\partial t} + \nabla \left( \alpha_k \rho_k \vec{V}_k \vec{V}_k \right) = -\alpha_k \nabla P_k + \nabla \alpha_k (\tau_k + \tau_k^t) + \alpha_k \rho_k \vec{g} + \vec{V}_{ik} \Gamma_k + \dot{M}_{ik} - \nabla \alpha_k \vec{\tau}_i \quad (29)
\]

where \( \alpha_k \) is the fraction of an element of volume occupied by component \( k \), \( \rho_k \) is the density of phase, \( \vec{V}_k \) is the velocity vector of a phase, \( P_k \) is the bulk pressure of a phase, \( \tau_k \) and \( \tau_k^t \) are the viscous and turbulent stress of a phase, \( \vec{V}_{ik} \) is the interfacial velocity vector, \( \Gamma_k \) is the mass generation, \( \vec{\tau}_i \) is the interfacial shear stress, and \( \dot{M}_{ik} \) interfacial momentum transfer term. The conservation of momentum requires

\[
\sum \dot{M}_{ik} = 0 \quad (30)
\]

This conservation of momentum term, the mass transfer effect force, turbulent stress, and the interfacial shear stress will be discussed in details in the following sections.

2.3 Interfacial Momentum Transfer Basset Force

The interfacial force is a sum of multiple forces, namely the interfacial drag, virtual mass, lift, diffusion, and Basset forces. The Basset force due to transient development of a boundary layer is given by Ishii and Chawla (1979) as

\[
\vec{F}_B = \frac{9}{2} \alpha_s \tau_s \sqrt{\rho_l \mu_m} \pi \int_t^D \left( \vec{V}_l - \vec{V}_s \right) \frac{d\xi}{\sqrt{t - \xi}} \quad (31)
\]

Neglecting the lift force due to rotation of particles and the diffusion force due to concentration gradient, a generalized force for a dispersed phase can be modeled as

\[
\dot{M}_{id} = \frac{\alpha_s \vec{F}_D}{B_s} + \frac{\alpha_s \vec{F}_v}{B_s} + \vec{F}_b \quad (32)
\]

where \( \vec{F}_D \) is the interfacial drag force, \( \vec{F}_v \) is the virtual or apparent mass force, \( B_s \) is the volume of a typical particle, and \( \vec{F}_b \) is the Basset force.

2.3.1 Interfacial Drag Force

The interfacial drag force which arrives from the viscous and pressure forces along the interface and is related to the local interfacial gradients is considered as the most important term in the interfacial momentum transfer. The interfacial drag force acting on a particle under steady-state condition can be written as

\[
F_D = \frac{1}{2} C_D \rho_l |V_r| A_{proj} \quad (33)
\]

where \( C_D \) is the drag coefficient, \( V_r \) is the relative velocity as \( V_r = V_s - V_l \) and \( A_{proj} \) is the projected area per unit volume of a typical solid particle and is given by Navickas et al. (1989)

\[
A_{proj} = 1.5 \frac{\alpha_s}{D_p} \quad (34)
\]

The drag coefficient for a multiparticle system depends on the flow regime and on the nature of the particles. For a solid-particle system the particle undergo Stokes, viscous, and Newton’s regime. In the viscous regime, a complete similarity exists between single and multiparticle systems if the Reynolds number was based on the mixture viscosity. However, for other regimes multiparticle correlations must be used. Therefore, for the different regimes the drag coefficient can be estimated as follows:
Prognosis and Numerical Analysis of the Pressure Drop

Stokes regime (Re\(_p\) ≪ 1)

\[ C_D = \frac{24}{Re_p} \]  
(35)

\[ Re_p = \frac{D_p \rho V_r}{\mu_m} \]  
(36)

The mixture viscosity can be calculated from the viscosity power law as

\[ \mu_m = \mu_l \left(1 - \frac{\alpha_s}{\alpha_{sm}}\right)^{-2.5\alpha_{sm}} \]  
(37)

For a solid-particle system, \(\alpha_{sm}\) is the maximum packing and is equal to 0.62 and the equation for mixture viscosity becomes

\[ \mu_m = \mu_l \left(1 - \frac{\alpha_s}{0.62}\right)^{-1.55} \]  
(38)

Viscous regime (Re\(_p\) ≪ 1000)

\[ C_D = \frac{24}{Re_p} \left(1 + 0.1Re_p^{3/4}\right) \]  
(39)

Newton’s regime (Re\(_p\) > 1000)

\[ C_D = 0.45 \left\{ \frac{1 + 17.67 [f(\alpha_s)]^{6/7}}{18.67 f(\alpha_s)} \right\} \]  
(40)

\[ f(\alpha_s) = \sqrt{1 - \alpha_s} \left(\frac{\mu_l}{\mu_m}\right) \]  
(41)

2.3.2 Virtual Mass Force

The virtual mass force \(F_v\) is a force that arises from the virtual increase of the solid-particle mass due to the small mass of liquid carried by the solid particle. This virtual mass force is equal to the mass of liquid carried by the solid sphere multiplied by a term that represents suitably the relative acceleration between phases, as

\[ F_v = \rho_l C_{VM} V_s A_{rel} \]  
(42)

where \(C_{VM}\) is the virtual mass coefficient, \(V_s\) is the solid sphere volume, and \(A_{rel}\) is the relative acceleration between the phases. This work is going to adopt the model of Ishii and Mishima (1984), where they expressed the virtual mass force per unit volume as

\[ F_v = \rho_l C_{VM} \left[\frac{D_s}{Dt} (V_r) - V_r \cdot \nabla V_l\right] \]  
(43)

where the material derivative \(D_s/Dt\) and the virtual mass coefficient are defined as

\[ \frac{D_s}{Dt} = \frac{\partial}{\partial t} + (V_s \nabla) \]  
(44)

\[ C_{VM} = \frac{1 + 2\alpha_s}{2(1 - \alpha_s)} \]  
(45)

Under steady-state conditions the virtual mass force becomes

\[ F_v = \frac{1}{2} \rho_l \frac{1 + 2\alpha_s}{1 - \alpha_s} [V_s \cdot \nabla V_r - V_r \cdot \nabla V_l] \]  
(46)
2.3.3 Mass Transfer Effect Force

The term \( V_{ik} \Gamma_k \) represents the mass transfer effect force, where \( V_{is} \) is the slip velocity because the velocity at the interface is different from the bulk velocity of a phase. The interfacial velocities are equal, in other words, \( V_{il} = V_{is} = V_i \). And, \( \Gamma_k \) is the mass generation term; it is equal to the rate of solid disappearance of the rate of liquid appearance. Therefore, the mass transfer effect force at the liquid and the solid interface, respectively, can be written as

\[
F_{ml} = (V_i - V_l) m_{ls} \quad (47)
\]

\[
F_{ms} = (V_s - V_i) m_{sl} \quad (48)
\]

\[
F_m = F_{ml} + F_{ms} = V_r m_{sl} \quad (49)
\]

2.3.4 Reynolds or Turbulent Stress

The term \( \tau_{ik} \) represents the turbulent stress or the Reynolds stress. This term can be estimated by time averaging the single-phase equations of mass and momentum. Seeking for this term, the continuity and the Navier–Stokes equations for a single-phase incompressible flow are expressed as

\[
\frac{\partial u_i}{\partial x_i} = 0 \quad (50)
\]

\[
\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial t_{ij}}{\partial x_j} \quad (51)
\]

where \( u_i \) and \( x_i \) are the instantaneous velocity and position, \( t \) is the time, \( P \) is the bulk pressure, \( \rho \) is the density, and \( t_{ij} \) is the viscous stress tensor defined by

\[
t_{ij} = 2\mu s_{ij} \quad (52)
\]

where \( \mu \) is the viscosity and \( s_{ij} \) is the strain-rate tensor and expressed as

\[
s_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (53)
\]

After time averaging these two equations, the Navier–Stokes equations become

\[
\rho \frac{\partial U_i}{\partial t} + \rho U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( 2\mu S_{ji} - \rho u'_j u'_i \right) \quad (54)
\]

where \( U_i \) and \( u_i \) are the mean and the fluctuation velocity and given by

\[
u_i (x, t) = U_i (x) + u'_i (x, t) \quad (55)
\]

\[
U_i (x) = \frac{1}{T} \int_{t}^{t+T} u'_i (x, t) \, dt \quad (56)
\]

The term \( \rho u'_j u'_i \) is known as the Reynolds-stress tensor and is denoted by \( \tau_{ij} \) or \( \tau_{ij}^{ren} \) in the below momentum equation, \( S_{ji} \) is the mean strain-rate tensor. This tensor is symmetric and thus has six independent components. Furthermore, this model adopts the two-equation \( k-\epsilon \) model. A two-equation turbulent model predicts the properties of a given turbulent flow by solving two equations; the Boussinesq approximation is the starting point of every two-equation model and is given by

\[
\tau_{ij} = 2\mu_T S_{ij} - \frac{2}{3} \rho k \delta_{ij} \quad (57)
\]

where \( \delta_{ij} \) represents the Kronecker delta and equals

\[
\delta_{ij} = \begin{cases} 
0 & \text{if } i \neq j \\
1 & \text{if } i = j 
\end{cases} \quad (58)
\]
The k-ε model is the most used two-equation model; k represents the kinetic energy per unit mass of the turbulent fluctuation as
\[ k = \frac{1}{2} \bar{u}' \bar{v}' = \frac{1}{2} (\bar{u}^2 + \bar{v}^2 + \bar{w}^2) \]  
(59)

where \( u' \), \( v' \), and \( w' \) are the fluctuating velocity components in the x, y, and z directions. This kinetic energy can be written in terms of density and eddy viscosity as
\[ \mu_T = C \rho k^{1/2} l \]  
(60)

\( C \) is a constant and \( l \) is a turbulence length scale. These equations lead to the following Reynolds stress tensor:
\[ \tau_{ij} = -\rho \bar{u}' \bar{v}' = -2 \rho k \]  
(61)

The quantity \( \varepsilon \) is the dissipation per unit mass and is defined by the following correlation where \( \nu \) is the kinematic viscosity:
\[ \varepsilon = \nu \frac{\partial u_i' \partial u_i'}{\partial x_k \partial x_k} \]  
(62)

Finally, without discussing the derivation details, the standard k-ε model follows multiple correlations from Wilcox (1993):
Eddy viscosity
\[ \mu_T = \rho C_k k^2 / \varepsilon \]  
(63)

Turbulence kinetic energy
\[ \rho \frac{\partial k}{\partial t} + \rho U_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial U_i}{\partial x_j} - \rho \varepsilon + \frac{\partial}{\partial x_j} \left[ (\mu + \mu_T / \sigma_k) \frac{\partial k}{\partial x_j} \right] \]  
(64)

Dissipation rate
\[ \rho \frac{\partial \varepsilon}{\partial t} + \rho U_j \frac{\partial \varepsilon}{\partial x_j} = C_{\varepsilon 1} \frac{\varepsilon}{k} \tau_{ij} \frac{\partial U_i}{\partial x_j} - C_{\varepsilon 2} \frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j} \left[ (\mu + \mu_T / \sigma \varepsilon) \frac{\partial \varepsilon}{\partial x_j} \right] \]  
(65)

Closure coefficients
\[ C_{\varepsilon 1} = 1.44, \quad C_{\varepsilon 2} = 1.92, \quad C_k = 0.09, \quad \sigma_k = 1.0, \quad \sigma \varepsilon = 1.3 \]

Auxiliary relations
\[ \omega = \varepsilon / C_k k \]  
(66)
\[ \ell = C_k k^{3/2} / \varepsilon \]  
(67)

### 2.4 Interfacial Shear Stress

This analysis assumes that the interfacial stress and the particle stress are equal, in other words
\[ \tau_s = \tau_{s,i} = \tau_{l,i} \]  
(68)

Drew (1978) created a model to link between the shear stress at the interface and the shear stress of the liquid where the liquid viscous stress is multiplied by a coefficient which is a function of solid fraction as follows:
\[ \tau_{s,i} = \beta(\alpha_s) \tau_l \]  
(69)

where \( \beta(\alpha_s) \) is the mobility factor and is given by the following relation:
\[ \beta(\alpha_s) = \frac{\mu_{eff}}{\mu_l} = \left( 1 + \frac{5}{2} \alpha_s \right) + \frac{9}{4 \{1 + [(1/2)(b/a)]\}} \left[ \frac{1}{(b/a)} - \frac{1}{[1 + (b/a)]} - \frac{1}{[1 + (b/a)]^2} \right] \]  
(70)
where \( b/a \) is given for a simple cubic packing by the following ratio:

\[
\frac{b}{a} = 2 \left\{ \frac{\left[ 1 - \left( \frac{\alpha_s}{\alpha_{sm}} \right)^{1/3} \right]}{\left( \frac{\alpha_s}{\alpha_{sm}} \right)^{1/3}} \right\}
\]  

(71)

Finally, for a 2D steady-state solid–liquid mixture the momentum equation for liquid and solid phase, respectively, can be written as

\[
\nabla (\alpha_l \rho_l V_l \tau_l) = -\alpha_l \nabla P_l + \nabla \left[ \alpha_l \left( \tau_{avg} + \tau_{avg}^l \right) \right] - \alpha_l \rho_l g + (V_l - V_i) \nabla \frac{\alpha_s F_D}{B_s} + \frac{\alpha_s F_v}{B_s} - \nabla \alpha_i \tau_i
\]

(72)

\[
\nabla (\alpha_s \rho_s V_s) = -\alpha_s \nabla P_s + \nabla \left[ \alpha_s \left( \tau_{avg} + \tau_{avg}^s \right) \right] - \alpha_s \rho_s g + (V_i - V_s) \nabla \frac{\alpha_s F_D}{B_s} - \frac{\alpha_s F_v}{B_s} - \nabla \alpha_s \tau_i
\]

(73)

By summing these two terms, we can obtain the general momentum equation:

\[
\nabla (\alpha_l \rho_l V_l \tau_l) + \nabla (\alpha_s \rho_s V_s) = V_i \Gamma - \alpha_l \nabla P_l - \alpha_s \nabla P_s + \nabla \left[ \alpha_l \left( \tau_{avg} + \tau_{avg}^l \right) \right] + \nabla \left[ \alpha_s \left( \tau_{avg} + \tau_{avg}^s \right) \right] - g (\alpha_l \rho_l + \alpha_s \rho_s) - \nabla \alpha_i \tau_i - \nabla \alpha_s \tau_i.
\]

(74)

### 2.5 Energy Conservation

The energy equations for a two-dimensional, two-phase, and steady-state flow are given for each phase separately by

\[
\nabla \left( \alpha_l V_l H_l \rho_l \right) = \alpha_l V_l \nabla P_l - Q_{wl} - Q_{il}
\]

(75)

\[
\nabla \left( \alpha_s V_s H_s \rho_s \right) = \alpha_s V_s \nabla P_s - Q_{ws} - Q_{is}
\]

(76)

where \( H \) is the specific enthalpy of a phase, \( Q_i \) is the interfacial heat transfer, and \( Q_w \) is a phase to environment heat transfer.

### 2.6 Phase to Environment Heat Transfer

#### 2.6.1 Vacuum-Jacketed Insulation

The primary source of heat transfer across vacuum-jacketed pipes is radiation; thus, solid conduction and gaseous convection are eliminated. The radiant heat transfer across vacuum-insulated pipes is given by a modified Stefan–Boltzmann equation:

\[
Q_w = F_e F_{1-2} \sigma A_1 \left( T^4_2 - T^4_1 \right)
\]

(77)

where \( \sigma \) is the Stefan–Boltzmann constant\(^1\), \( A_1 \) is the area of surface 1, \( T \) is the bulk temperature, \( F_e \) is the emissivity factor, \( F_{1-2} \) is the configuration factor, subscript 1 refers to the enclosed or inner surface, and subscript 2 refers to the outer surface.

Without radiation shields:

\[
\frac{1}{F_e} = \frac{1}{\varepsilon_1} + \frac{A_1}{A_2} \left[ \frac{1}{\varepsilon_2} - 1 \right]
\]

(78)

With radiation shields:

\[
\frac{1}{F_e} = \left[ \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_s} - 1 \right] + (N - 1) \left[ \frac{2}{\varepsilon_s} - 1 \right] + \left[ \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_s} - 1 \right]
\]

(79)

where \( N \) is the number of shields, \( \varepsilon \) is the emissivity, and \( \varepsilon_s \) is the emissivity factor for a shield emissivity (parallel flat plates). Ewart and Dergance (1978)\(^2\) stated that actual values of the emissivity factor are approximately five times greater than the calculated values in the above equations\(^2\). Therefore, the calculated values will be multiplied by a factor of 5 in order to produce more precise values. For cryogenic vessels, the inner vessel is completely enclosed by the outer vessel, therefore, \( F_{1-2} = 1 \). Figure 1 shows a cross section of the vacuum-jacketed transfer line.

\(^1\)\( \sigma = 56.69 \text{ nW/m}^2\text{K}^4 \)

\(^2\)Multiply the resulting emissivity factors from Eqs. (78) and (79) by 5.
2.6.2 Standard Insulation Systems

The heat transfer across standard insulation system can be written as

$$Q_w = UA_0(T_{air} - T_{fluid})$$

(80)

$$\frac{1}{UA_0} = \frac{1}{h_iA_i} + \frac{\Delta r_n}{k_n A_{n,lm}} + \frac{1}{h_0 A_0}$$

(81)

where $n$ is a subscript referring to type of material or types of insulation, $A_i$ and $A_0$ are the inside and outside surface area of a pipe, respectively, given by

$$A = \pi DL$$

(82)

The area $A_{n,lm}$ is called the log-mean area and defined as

$$A_{lm} = \frac{2\pi (r_0 - r_i)}{\ln(r_0/r_i)}$$

(83)

Finally, $h_i$ is given by Eqs. (19) and (20) for each of the phases $h_0$ is the outside heat transfer coefficient due to air at 25°C. This empirical equation is given by Churchill and Bernstein (1977)

$$h_0 = \frac{k_{air}}{d} \left\{0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5}\right\}$$

(84)

The wall boundary condition for the pipe and the fitting is

$$-k_s \frac{\partial T}{\partial y} \bigg|_{y=R} = q''_w$$

(85)

where $q''_w$ is the surface flux. Figure 2 shows a cross section of the standard insulation line.
3. STUDY CASE

The model consists of five schedule 5S 1.5 in. straight sections pipes each 6.1 m (20 ft.) in length, five bellows each 0.3048 m (1 ft.) in length, 4 bellows each 0.24 m (0.79 ft.) in length, and two globe valves 0.1524 m (0.5 ft.) in length; the valves are placed 0.3048 (1 ft.) away from the inlet and 1 ft. away from the outlet and connected to two 1 ft. sections of pipes. At each end of the 20 ft. section of a pipe, a bellow is placed inside the pipe and a bayonet is used to connect one section to another. Figures 3–6 represent different sections of the above described transfer line, whereas Fig. 7 describes the cross section of the line. The line is vacuum-insulated with 10 radiation shields with the emissivity values for polished stainless steel where, $\varepsilon_1 = \varepsilon_2 = 0.3$ and $\varepsilon_s = \varepsilon_{\text{shield}} = 0.03$. These values are used to calculate the heat flux across the wall boundaries in each type of element and in addition to calculate pressure loss across the two valves. The geometry is created using ANSYS DesignModeler (ANSYS, 2016). The valves, bellows, and bayonets are modeled as straight pipe sections where pressure drops and heat leaks are added to each of these components in the setup.

The mesh is generated with advanced sizing function on; a body sizing is also added where the element size is set equal to 2 mm. An additional feature is added to get more accurate results for the boundary layers and for the core of the pipe. The mesh results in 728,402 hexahedron elements, which gives 728,402 nodes for a meter pipe. Figure 8 shows the mesh generated of a straight pipe length.

The most adequate model that fits slush hydrogen modeling is the Eulerian–Eulerian model in which both phases are treated as an Eulerian phase. The interfacial terms can be solved using the particle model, mixture model, and
**FIG. 4:** End of a 20 ft. section pipe including a bellow and a bayonet

**FIG. 5:** External diameter of two pipe sections connected by a bayonet

**FIG. 6:** External diameter of two pipe sections connected by a bayonet
the free surface model; the model is chosen based on the type of the multiphase flow. In the slush hydrogen case, liquid hydrogen is considered as the continuous fluid and the solid hydrogen is considered as the dispersed solid. In such case, the particle model is chosen to best fit the flow. In addition, the flow must be distinguished between homogeneous and inhomogeneous, where the homogeneous flow model assigns one velocity field to the flow and the inhomogeneous flow model treats the two phases as separated and assign different properties to each of the phases.

First, since the properties of hydrogen at the triple point are not implemented into CFX (computational fluid dynamics software tool similar to ANSYS Fluent), two materials were created, one describing the solid hydrogen properties at triple point and the other describing liquid hydrogen properties at triple point. Another homogeneous binary material was created to define saturation properties of liquid and solid hydrogen. The latter is going to be used in the phase-change modeling. Properties such as viscosity, density, molar weight, thermal conductivity, thermal expansivity, and reference state enthalpies were found in Reynier and Bugel (2011) and Roder (1977) and introduced into the materials section, respectively, for solid and liquid hydrogen. The fluid or solid regions are called domains in CFX; a fluid domain is created and located at the region inside the inner pipe along the whole line. Then, the two created materials, solid and liquid hydrogen, are now set into the fluid and particles definition to define the fluid domain. Liquid hydrogen is set as the continuous phase and solid hydrogen is set as the dispersed phase with a mean diameter of 2 mm and a maximum packing as 0.62. The mostly used thermodynamic properties at triple point are presented in Table 1.

The second step is to choose the inhomogeneous model and the type of heat transfer equation to be treated. CFX offers various heat transfer models and solves a different equation for each model chosen. Thermal energy and total energy are the two equations that suit the model where the thermal energy equation deals with the static enthalpy and is used for incompressible and low-speed compressible flows only, and the total energy equation deals with the total

---

**FIG. 7:** The outer pipe modeling the vacuum space

**FIG. 8:** Mesh of a straight pipe section

| Interfacial Phenomena and Heat Transfer |

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TABLE 1: Thermodynamic properties of hydrogen around triple point

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Solid</th>
<th>Liquid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molar mass</td>
<td>kg/kmol</td>
<td>2.016</td>
<td>2.016</td>
</tr>
<tr>
<td>Density</td>
<td>kg/m³</td>
<td>86.5</td>
<td>77.04</td>
</tr>
<tr>
<td>Specific heat capacity</td>
<td>J/(kg.k)</td>
<td>2865</td>
<td>6360</td>
</tr>
<tr>
<td>Ref. temperature</td>
<td>K</td>
<td>13.8</td>
<td>13.81</td>
</tr>
<tr>
<td>Ref. pressure</td>
<td>kPa</td>
<td>—</td>
<td>7.04</td>
</tr>
<tr>
<td>Ref. spec. enthalpy</td>
<td>J/kg</td>
<td>−367,180</td>
<td>−308,900</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>kg/(m.s)</td>
<td>1.00E−15</td>
<td>2.58E−05</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>W/(m.K)</td>
<td>0.9</td>
<td>0.073</td>
</tr>
<tr>
<td>Thermal expansivity</td>
<td>K’−1</td>
<td>—</td>
<td>0.0102</td>
</tr>
</tbody>
</table>

enthalpy and is used for high-speed flows. The thermal and total energy equations for a steady state are, respectively, shown as

\[
\nabla \left( \alpha_k V_k H_{k,tot} \rho_k \right) = \nabla \left( \alpha_k k_k \nabla T_k \right) + \alpha_k \tau_k : \nabla V_k + S_{E_k} + Q_k + \sum \left( \Gamma H_{ki,tot} - \Gamma H_{ik,tot} \right) \tag{86}
\]

\[
\nabla \left( \alpha_k V_k H_k \rho_k \right) = \nabla \left( \alpha_k k_k \nabla T_k \right) + \alpha_k \tau_k : \nabla V_k + S_{E_k} + Q_k + \sum \left( \Gamma H_k - \Gamma H_{ik} \right) \tag{87}
\]

where \( H_{k,tot} \), \( k \) and \( H_k \) represent the total enthalpy, thermal conductivity, and static enthalpy. \( S_{E_k} \) describes an external heat source and the term \( \sum (\Gamma H_k - \Gamma H_{ik}) \) is the heat transfer induced by interfacial mass transfer. As for turbulence, the \( k-\varepsilon \) model is chosen for the continuous phase with the parameters specified in Section 3 and the dispersed phase zero equation model is used for the solid phase, where

\[
\nu_{td} = \frac{\nu_{tc}}{\sigma} \tag{88}
\]

\[
\mu_{td} = \frac{\mu_{tc} \rho_d}{\sigma \rho_c} \tag{89}
\]

where \( \sigma \) is the turbulent Prandtl number, \( \nu_{td} \) and \( \nu_{tc} \) are the dispersed and continuous kinematic eddy viscosity, respectively.

The Schiller Naumann drag model is used for particle models and the interfacial drag coefficient is computed from the following equation:

\[
C_D = \frac{24}{Re} \left( 1 + 0.15Re^{0.687} \right) \tag{90}
\]

The interphase mass transfer is the phenomenon responsible for the solid volume fraction loss. In this case the mass transfer happens by thermal phase change where a part of the solid phase melts and transforms into liquid hydrogen. The saturation properties for hydrogen are 13.8 K and 7.04 kPa. The mass transfer is related to the interphase heat transfer, whereas the heat transfer model related to thermal phase change is the two resistance model, where the two heat transfer equations from each phase are

\[
Q_{il} = h_{il} A_i (T_i - T_{il}) \tag{91}
\]

\[
Q_{ls} = h_{ls} A_s (T_s - T_{il}) \tag{92}
\]

where \( h_i \) is the interfacial heat transfer coefficient and \( T_i \) is the interfacial temperature and is set equal to saturation temperature by the solver. Then, the thermal phase change equation is determined from the total heat flux balance as

\[
m_{ts} = \frac{Q_{il} + Q_{ls}}{H_{fusion}} \tag{93}
\]
The correlations used by CFX to determine the interfacial heat transfer coefficient are the Ranz and Marshall (1952) and Hughmark (1967) methods; in addition, a zero-resistance option can be chosen on the dispersed phase to force the bulk temperature of the phase to stay equal to the saturation temperature. The equations for Ranz–Marshall and Hughmark are, respectively, shown as

\[ \text{Nu} = 2 + 0.6 \text{Re}^{0.5} \text{Pr}^{0.3} \] \hspace{1cm} (94) \]

\[ \text{Nu} = 2 + 0.6 \text{Re}^{0.5} \text{Pr}^{0.33} \] \hspace{1cm} (95) \]

\[ \text{Nu} = 2 + 0.6 \text{Re}^{0.62} \text{Pr}^{0.33} \] \hspace{1cm} (96) \]

In order to cover larger ranges of Reynolds and Prandtl number, two subroutines using CFX expression language were added to calculate the interphase heat transfer coefficients for each phase as derived in Section 2.

Setting boundary conditions for the inlet, outlet, and the walls is important in order to determine the upstream of the inlet pressure, the bulk mass flow rate and the heat leak through the pipe. Pressure of 241 kPa or 35 psia is found from earlier studies to be a good value for the upstream pressure to prevent stagnation of the flow; inlet temperatures are equal to the triple point and the solid fraction is equal to 0.5, which corresponds to 50% slush hydrogen. The wall heat fluxes across the pipe sections are calculated from the radiation equation for vacuum-jacketed pipes along with 10 radiation shields. Heat leaks for valves, bayonets, and bellows are found from Hardy (1990) for different materials.

The pressure loss inside the valves is modeled by adding a loss coefficient for a momentum source term. Flow coefficient, \( C_v \), or loss coefficient, \( k \), are easily found for a globe valve which is fully open. The isotropic loss model is solved by CFX by the following equation:

\[ S_M = -\frac{\mu}{K_{perm}} V - K_{loss} \frac{\rho}{2} |V| \] \hspace{1cm} (97) \]

where \( S_M \) is the momentum source, \( K_{perm} \) is the permeability and \( K_{loss} \) is the quadratic loss coefficient and can be thought as “pressure drop per unit head.”

4. RESULTS AND DISCUSSION

The interfacial phenomena, the effective multiparticle drag law, and turbulence effects allow the model to predict results for pressure drop more accurately. The advantage of using the multiparticle drag law over the single-particle drag law is that the multiparticle drag law predicts more accurately the pressure drop. In an earlier analysis, Zakhia (2001a) compared the pressure drop for both of these laws, where the multiparticle drag law results in much higher pressure drop and especially on high flow rates, where the flow eddies become more significant and thus results in a higher pressure drop. Multiparticle drag law predicted more accurate results in comparing with experimental data and this is due to the higher contribution of interfacial forces in the linear momentum equation. Permission had been gained from the author to use and compare his results with the obtained results in this work to ensure the validation of this model. In addition, the results are compared with experimental data from NASA, Lewis Research Center, Cleveland, OH for vacuum-jacketed pipe of 1.5 in. in diameter. Furthermore, experimental data obtained by Sindt and Ludtke (1969) analyzing pressure drop for a flow loop, consisting of a 0.4 m\(^3\) slush hydrogen generator, a 24.4 m long vacuum-insulated transfer line, and a 0.5 m\(^3\) receiver dewar, are going to be shown and discussed. The results of the comparison made by Zakhia (2001d) between the two laws for different flow rates in a 1.5 in. vacuum-jacketed pipe is shown in Fig. 9.

The flow test completed by Sindt and Ludtke (1969) was used to determine pressure drops in the discussed flow loop for a fresh and an aged slush mixture. Slush was made in the generator by the freeze–thaw method and transferred to the receiver dewar through the transfer line by the means of a differential pressure, where the system was capable of delivering approximately 145 kPa total pressure differences. Tests were made on triple-point hydrogen in addition to mass solid fraction ranging from 0.18 to 0.5. Results showed that above 0.85 dm\(^3\)/s flow rate and solid fractions to 0.3 the losses were less than that of triple-point hydrogen. The results proved that the higher the solid fraction, the greater the pressure drop.

Interfacial Phenomena and Heat Transfer
For an aged mixture, the pressure loss is 4 to 10% above that of a fresh mixture. Sindt (1970) stated that this loss is probably due to the change in the solid-particle structure which occurred during aging. Several more studies were done to provide data for the pressure loss and solid volume fraction loss for fresh and aged mixtures. The model created in this work is equivalent to an existing transfer line at the NASA Lewis Research Centre K-site Facility.

At the K-site facility the slush hydrogen is produced by the evaporative cooling method or the freeze–thaw method inside a 4.92 m$^3$ (1300 gal.) vacuum-insulated slush hydrogen generator. Following the production, slush hydrogen was transferred to a spherical test tank. The transfer line was 38.1 m (125 ft.) long and constructed of a 1.5 in. Schedule 5S stainless steel vacuum-jacketed pipe in addition to five valves, elbows, bayonet fittings, and bellows. Densimeters were installed to measure density along with pressure sensor in the line to determine the upstream line pressure and the pressure measurement for the spherical test tank. Results for pressure drops for triple-point, normal boiling point, and slush hydrogen are shown in Figs. 10, 11, and 12 in addition to the solid fraction loss.

**FIG. 9:** Comparison of pressure drops for two laws vs. flow rate for a 1.5 in. vacuum-jacketed line, 50% slush inlet

**FIG. 10:** Pressure contour at 10 gpm
Zakhia (2001a) predicted pressure drops and solid volume fraction drop in his analysis by using and modifying PHOENICS software. The predictions are made for flows of a minimum of $6.30902 \times 10^{-5}$ m$^3$/s (10 gpm) to $0.01261804$ m$^3$/s (200 gpm) in increments of 5 gpm. This study predicted more accurate results than FLUSH, especially, for higher flow rates in which turbulence effects became significant. Zakhia (2001a) predicted results for 1.5 in. vacuum-jacketed and normal insulated pipes. In addition, the model predicted solid volume fraction loss for 1, 2, and 4 in. pipe diameter and concluded that at lower flow rates, heat leak from the environment dominates the solid fraction loss, while at higher flow rates friction heating is highly responsible for the solid hydrogen degradation. For each diameter an optimum flow rate was found, where the solid fraction loss becomes minimum. It is obvious that the solid fraction loss for normal insulated pipes is higher than that of vacuum-insulated pipes. It can be seen from the heat transfer equation that the heat leak into a normal insulated pipe is much higher than the heat leak into a vacuum-insulated pipe. This is to enforce that at low flow rates, since the heat leak from the environment dominates the fraction loss, the solid fraction loss at low flow rates in a normal insulated pipe increases dramatically if it is compared to a vacuum-insulated pipe.
In this paper, simulations were done to predict pressure drop in a 1.5 in. Schedule 5S vacuum-jacketed line in function of flow rate. Flow rates range from a minimum of 10 gpm to a maximum of 200 gpm, increasing 10 gpm with each simulation. The order of convergence was set to 10e-6 root-mean-square. Initial values were inserted into the simulation setup for pressure, temperature, and volume fractions. Twenty simulations were run in total, and, the results obtained were compared to the analysis done by Zakhia (2001a), and to the experimental data done at the K-site facility. As stated before, the analysis was based on the multiparticle drag law correlations to estimate the drag coefficients because this law showed a better fit to experimental data in earlier results.

It can be seen from the results that the pressure drop is greater at higher flow rates. This is due to the increased velocity because for the same pipe diameter the head loss takes the form of

$$H = \frac{f \rho V^2 L}{2D}$$

where $H$ is the head loss. Therefore, for the same pipe diameter and length the increased velocity will increase the head loss and eventually the pressure drop. Figure 13 shows the pressure drop vs. flow rate curve.

A comparison is made between the results of this analysis (Zakhia, 2001a), FLUSH (Hardy, 1990), and experimental data as shown in Fig. 14. The analysis shows good agreement with Zakhia, FLUSH, and the experimental data.
data at low flow rates, whereas, at medium flow rates, from 100 to 160 gpm, the results deviate from the experimental data but show better agreement than FLUSH, which uses a one-dimensional model. The results extracted by Zakhia (2001a) had better agreement at these flow rates because interfacial shear stress and solid pressure were added. In this analysis, the interfacial shear stress was not implemented into the simulation setup nor the solid pressure models which may affect the results because these models reflects the real aspects of the flow in different ways. At high flow rates, 170 to 200 gpm, this analysis shows better agreement with the experimental data. This is due to the 3D model which predicts data more accurately where turbulence effects become more significant. However, since the experimental results fluctuate considerably, good comparison between models can be tough. Therefore, a value of the deviation is calculated for multiple flow rates and shown in Table 2. As interfacial shear stress and interfacial solid pressure is missing in this model, it can be seen that a 3D model alone would give better results. However, in order to get more accurate results, interfacial shear stress and solid pressure must be added.

<table>
<thead>
<tr>
<th>Flow rate (Gpm)</th>
<th>Deviation analysis</th>
<th>Deviation Zakhia</th>
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</thead>
<tbody>
<tr>
<td>120</td>
<td>0.487</td>
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<tr>
<td>180</td>
<td>0.0501</td>
<td>0.305</td>
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</tbody>
</table>

REFERENCES

Prognosis and Numerical Analysis of the Pressure Drop
