FILM FLOW WITH LOCAL HEATING: ANALYSIS OF 2D STRUCTURE INSTABILITY

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This paper is devoted to the analytical study of the stability of a two-dimensional (2D) structure of liquid film flow with local heating. The limiting irrotational 2D steady-state solution for a gravity-driven non-isothermal thin film (including the case with a moving heat source) is considered. The instability of this flow regime with respect to infinitesimal long-wavelength transverse perturbations is revealed, and the expression for the characteristic wavelength of unstable perturbation is derived. The dependence of a developing three-dimensional flow structure period on the main physical parameters is analyzed. Good accordance with the known experimental and theoretical data is obtained. The numerical solution to the quasi-linear evolution equation is presented.

KEY WORDS: *thin liquid film, flow, local heating, thermal capillarity, two-dimensional (2D) critical regime, linear analysis of stability, periodic structure, quasi-linear evolution equation, numerical solution*

1. INTRODUCTION

The novel two-dimensional (2D) and three-dimensional (3D) regimes of gravity-driven liquid flow with local heating were discovered and studied experimentally by Kabov et al. (1996a,b) and Kabov (1998). Thin film deformation due to the thermocapillary effect is of practical importance in order to provide appropriate thermal regimes for small electronic devices operating with high heat flux production and temperature non-uniformities rising up to 10 K/mm. Thermocapillarity can lead to local film thinning and rupture with device overheating. If the heat flux is below a certain limit, a 2D steady-state flow regime takes place. In a horizontal layer, gravity-driven fluid flow is absent and a dry spot can appear at the fixed heater. Under microgravity conditions even a low heat flux can result in this type of situation. Then, it is necessary to induce the relative motion of the liquid and heater by gas flow (shear-driven film) (Gatapova et al., 2004). In the case of a moving local heat source, film rupture can also be prevented due to the inertia of the liquid (Kuibin and Sharypov, 2013). This mechanism is able to replace the stabilizing effect of gravitation or gas flow. Nevertheless, for all of the aforementioned physical situations the existence of 2D steady-state liquid flow is limited: the 2D regime becomes unstable at a certain critical heat flux. Thus, the important problem is to predict the critical parameters and describe the development of the 3D flow structure.

The theoretical study of non-isothermal film dynamics has resulted in various evolution equations derived for different heat conditions (Kopbosynov and Pukhnachev, 1986; Oron and Rosenau, 1992; Joo et al., 1991; Miladinova et al., 2002). The case of a fixed local heater was studied by Skotheim et al. (2003), Kalliadasis et al. (2003), Marchuk and Kabov (1998), Sharypov et al. (2001), and Kuznetsov (2000). The 2D flow structure in a gravity-driven thin liquid film with a moving local heat release zone was theoretically studied by Sharypov et al. (2001). This type of scheme was realized in experiments with a combustion wave propagating along a thin metal substrate covered by a fuel film (Korzhavin et al., 1998).

2. CRITICAL REGIME OF 2D STEADY-STATE FLOW

The statement of the problem is the following. A thin film of an incompressible viscous liquid can flow down along a plate inclined at an angle θ with respect to the horizontal. A plane heat wave moves in a substrate along the x-axis (in the opposite direction to the liquid flow) at speed $C =$ const. The characteristic length of the temperature variation

NOMENCLATURE

C	velocity of the moving heat source (m/s)
g	gravity acceleration (m/s^2)
h	film thickness (m)
i	imaginary unit
L	characteristic length of the heating zone (m)
l_σ	capillary constant (m)
n	normal to the free surface
p	pressure (Pa)
Pr	Prandtl number
q	heat flux (W/m^2)
r	main radius of curvature of the free surface (m)
Re	Reynolds number
T	absolute temperature (K)
t	time (s)
u, v, w	components of the velocity (m/s)
x, y, z	spatial coordinates (m)

Greek Symbols

Γ	flow rate (kg/m · s)
γ	wave number (m^{-1})
ε	small parameter
η	dynamic viscosity (kg/m · s)
θ	inclination angle (rad)
κ	wave number (m^{-1})
Λ	wavelength (m)
λ	wave number (m^{-1})
μ	small parameter
ρ	density (kg/m^3)
σ	surface tension (kg/s^2)
$\hat{\sigma}$	tensor of the viscous stresses (Pa)
φ	dimensionless coordinate
Ω	increment of perturbation (s^{-1})

Superscripts

g	gas phase
$*$	complex conjugate
$'$	perturbation of the two-dimensional steady-state solution

Subscripts

cr	critical condition
j	harmonic number
n	normal vector component
s	perturbation wave mode number
t, x, y, z, φ	derivation
$*$	unstable linear perturbation with maximal amplification rate
∞	value of parameters far from temperature inhomogeneity

zone is $L = L_+ + L_-$ (see Fig. 1), which is assumed to be much larger than the thickness of the undisturbed layer (h_∞) far from the heating zone: $h_\infty/L = \mu \ll 1$. The heating induces a thermal boundary layer, which reaches the free surface and causes a temperature gradient at the free surface. The thermocapillary force is directed opposite to the flow and deforms the flat film surface. We assume that the characteristic streamwise length scale of the deformation occurring at the surface has an order of magnitude L. Furthermore, this length scale is assumed to be much greater than the deformation amplitude: $|\partial h(t,x)/\partial x| = O(\mu)$, where h is the layer thickness; t, x, y, and z are the time and spatial variables, respectively; the y-axis is normal to the substrate; and **g** is the gravity acceleration. The latter allows using the long-wave approximation (Alekseenko et al., 1994; Craster and Matar, 2009; Kalliadasis et al., 2012) in the general governing equations (i.e., continuity, Navier-Stokes, and heat transfer equations).

Both the viscosity and density of the liquid are assumed to be independent of the temperature: η = const, ρ = const. The heat, mass, and momentum exchanges with the gas phase are neglected. The frame of reference is connected to the heat release zone; therefore, the velocity at the solid boundary is $(u + \mathrm{C}) = v = w = 0$, $y = 0$. At the free surface the kinematic condition is satisfied: $v = \mathrm{d}h/\mathrm{d}t = h_t + uh_x + wh_z$, $y = h$, and the balance of stresses can be written as follows (Landau and Lifshits, 1987):

$$[p - p^g - \sigma\,(1/r_1 + 1/r_3)]\, n_i = (\hat{\sigma}_{ik} - \hat{\sigma}^g_{ik})\, n_k + \partial\sigma/\partial x_i, \quad y = h$$

where p is the pressure; u, v, and w are the fluid velocity components; σ is the surface tension; $\sigma(x) = \sigma_\infty + (d\sigma/dT\,[T(x) - T_\infty])|_{y=h}$, with $(d\sigma/dT)$ = const; T is the temperature of the liquid; r_1 and r_3 are the main radii of

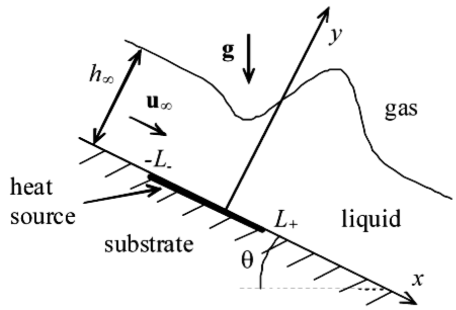

FIG. 1: Scheme of 2D film flow

curvature of the free surface in the x- and z-direction, respectively; $\hat{\sigma}_{ik}$ are tensor components of the viscous stresses; and n_i are the components of the vector normal to the free surface, i.e., $\mathbf{n} = \mathbf{x}n_1 + \mathbf{y}n_2 + \mathbf{z}n_3$, $n_{1,3} = O(\mu)$, and $n_2 = O(\mu^0)$. The superscript g hereafter refers to the gas phase (p^g = const; $\hat{\sigma}^g = 0$); subscripts t, x, y, and z denote derivation; and subscript ∞ denotes conditions far from the heating zone. The normal component of the heat flux is equal to zero at the free surface:

$$|\mathbf{q}_n| \sim |\partial T/\partial n| = |h_x T_x - T_y + h_z T_z| \cdot \left[h_x^2 + 1 + h_z^2\right]^{-1/2} = 0, \quad y = h$$

The other thermal boundary conditions are: $T\,(x \to -\infty) = T_\infty$ = const, $T\,(-L_- > x, y = 0) = T\,(x > L_+, y = 0)$ = T_∞, $T\,(-L_- \leq x \leq L_+, y = 0) = T_\infty$, or $|\partial T/\partial y| \sim q_w \neq 0$ as $-L_- \leq x \leq L_+$, y = 0.

In the 2D steady-state regime the following expressions can be written (Sharypov and Kuibin, 2008):

$$\begin{aligned} p &= p^g + (h-y)\,\rho\,|\mathbf{g}|\cos\theta - \sigma h_{xx} \\ u &= -C + y\sigma_x\eta^{-1} + \left(y^2/2 - hy\right)\{\cdots\}\,\eta^{-1} \\ v &= \left[y^2 h_x\{\cdots\} + y^2(h - y/3)\{\cdots\}_x - y^2\sigma_{xx}/\rho\right](2\eta)^{-1} \end{aligned} \tag{1}$$

where $\{\cdots\} \equiv \{h_x\rho\,|\mathbf{g}|\cos\theta - \rho\,|\mathbf{g}|\sin\theta - (\sigma h_{xx})_x\}$. The convective terms are omitted due to the assumption of the low Reynolds number: $\mathrm{Re}_\Gamma = |\Gamma|/\eta = \left|h_\infty^3\rho^2\,|\mathbf{g}|\sin\theta/\left(3\eta^2\right) - \rho C h_\infty/\eta\right|$. The condition of constant flow rate, $\Gamma = \rho\int_0^h u dy = \rho^2\,|\mathrm{g}|\sin\theta h_\infty^3/\,(3\eta) - \rho C h_\infty$ = const, results in the following equation for the film thickness:

$$\left(\frac{h^3}{h_\infty^3} - 1\right)\sin\theta + \frac{h^3}{h_\infty^3}\left\{\frac{\sigma h_{xxx}}{\rho\,|\mathbf{g}|} - h_x\cos\theta\right\} + \frac{3h^2\sigma_x}{2\rho\,|\mathbf{g}|\,h_\infty^3} = \frac{3\eta C}{\rho\,|\mathbf{g}|\,h_\infty^2}\left(\frac{h}{h_\infty} - 1\right). \tag{2}$$

The thickness of the layer depends on σ_x, which is determined from the conjugated hydrodynamic and thermal problem; this was numerically studied in Kuibin and Sharypov (2013) for different values of and θ. The alternative approach is to use the known distribution $T\,(x,\,y = h)$ (Sharypov and Kuibin, 2009b), which can be measured experimentally and approximately described as $\sigma_x = (d\sigma/dT)\,T_x|_{y=h} = -\,|\sigma_x|_{\max}\cdot\exp\left[-\,(x/L_\pm)^2\right]$ in the regions $L_+ > x > 0$ and $0 > x > -L_-$, respectively (in this case hydrodynamic and thermal problems are connected by the boundary condition only, thus it is not necessary to solve the thermal problem).

From Eq. (2) it follows that there exists a critical value $(\sigma_x)_{\mathrm{cr}}$, when the minimal velocity at the free surface is equal to zero (let it be at point $x = 0$). If $|\sigma_x| > |\sigma_x|_{\mathrm{cr}}$, then a zone with reverse flow appears, which means a vortex zone with closed streamlines is present (Sharypov and Kuibin, 2009a). In the case, $\tan\theta \gg |h_x|$, $C = 0$, and neglecting surface pressure σh_{xx}, we can obtain the following equations from Eqs. (1) and (2): $u\,(0,h) = 0$, $h_{\max} = h(0) = 2^{2/3}h_\infty$, $h_x(0) = 0$, and $\sigma_x(0) = (\sigma_x)_{\mathrm{cr}} = -2^{-1/3}\rho\,|\mathbf{g}|\,h_\infty\sin\theta$. The numerical solution (Sharypov and Medvedko, 2000) to Eq. (2) yields more precise values: $h_{\max} = 1.47h_\infty$ and $(\sigma_x)_{\mathrm{cr}} = -0.92\rho\,|\mathbf{g}|\,h_\infty$, which are close to the previously written approximate analytical estimations. The calculations correspond to the conditions of the experiments (Kabov, 1999) with (25% C_2H_5OH + 75% H_2O), $C = 0$, $\theta = \pi/2$, $T_\infty = 303$ K, $\mathrm{Re}_\Gamma = 2$, Prandtl number Pr = 14.7, $\rho = 956$ kg/m^3, $\eta/\rho = 1.8 \times 10^{-6}$ m^2/s, $\bar{\sigma}_\infty \equiv \sigma_\infty \cdot \left(\rho h_\infty^2 \left|\mathbf{g}\right| \sin\theta\right)^{-1} \approx 230$, $d\sigma/dT = -1.1 \times 10^{-4}$ kg/(s^2 · K), $l_\sigma \approx (l_\sigma)_\infty = 1.91 \times 10^{-3}$ m, $h_\infty \approx 1.26 \times 10^{-4}$ m, $u|_{x\to-\infty, y=h_\infty} = h_\infty^2 \rho \left|\mathbf{g}\right| \sin\theta/(2\eta) \approx 4.3 \times 10^{-2}$ m/s, and $L_+ = 3L_- = 12h_\infty$. These values are also used subsequently.

At point $x = 0$ we also have the following: $\sigma_{xx} = 0$, $u_x = 0$, $v = 0$, $v_y = 0$, $v_x = 0$, and $T_{xx}|_{y=h} = 0$. Thus, we can obtain a number of estimations for the magnitude of all of the terms in the equations and boundary conditions (for the 2D steady-state problem) in the region $|x| \le \mu L_\pm \approx h_\infty$, and use them to simplify the analysis of the stability (Sharypov and Medvedko, 2000).

3. LOCAL STABILITY ANALYSIS

Let us assume that the 2D steady-state solution (obtained for the critical regime) is perturbed in the region $|x| \le \mu L$ and the amplitude of the perturbation has the order of magnitude $O(\varepsilon) = O(\mu^2)$. Neglecting the terms $\sim O(\varepsilon^2)$ in the governing equations and boundary conditions at the perturbed free surface, we can seek the solution in the following form:

$$H' = \sum_{s=1}^{S} \left[H'_s + (H'_s)^*\right], \quad \sigma' = \sum_{s=1}^{S} \left[\sigma'_s + (\sigma'_s)^*\right], \quad h' = \sum_{s=1}^{S} \left[h'_s + (h'_s)^*\right]$$

where $H' \equiv \{u', v', w', p'\}$ are the perturbations of hydrodynamic parameters; superscript * denotes the complex conjugate; and subscript s denotes the wave mode of perturbation, where each mode is represented by superposition of periodic harmonics in the z-direction:

$$\begin{aligned} H'_s(x,y,z,t) &= \sum_{j=1}^{\infty} \left[H'_s(y)\right]_j \exp\left(\Omega_j t + i\kappa_j z + \lambda_j x\right) \\ \sigma'_s(x,z,t) &= \sum_{j=1}^{\infty} \left(\delta\sigma_s\right)_j \exp\left(\Omega_j t + i\kappa_j z + \lambda_j x\right) \\ h'_s(x,z,t) &= \sum_{j=1}^{\infty} \left(\delta h_s\right)_j \exp\left(\Omega_j t + i\kappa_j z + \lambda_j x\right) \end{aligned} \tag{3}$$

where κ and λ are wave numbers; Ω is the complex increment; $\mathrm{Im}(\kappa_j) = 0$; and $i = \sqrt{-1}$.

The problem leads to the well-known Orr–Sommerfeld equation, $\mathrm{Im}(\lambda) = 0$, because the perturbations are not periodic in the x-direction. We also assume the perturbation amplitude is independent of the x-coordinate, $\mathrm{Re}(\lambda) = 0$. After these assumptions, we obtain the linear ordinary differential equation with constant coefficients for the dimensionless perturbation of the velocity y-component, which is different from that obtained in Sharypov and Medvedko (2000):

$$\bar{v}'_{\varphi\varphi\varphi\varphi} - \bar{v}'_{\varphi\varphi}\left(\bar{\Omega} + 2\mu^2\bar{\kappa}^2\right) + \bar{v}'\left(\bar{\Omega}\mu^2\bar{\kappa}^2 + \mu^4\bar{\kappa}^4\right) = 0 \tag{4}$$

where $\bar{v}' = v'_j \rho h_\infty/\eta$, $\varphi = y/h_\infty$, $\bar{\kappa} = \kappa_j L$, and $\bar{\Omega} = \Omega_j \rho h_\infty^2/\eta$. Equation (4) with the boundary condition $\bar{v}'|_{\varphi=0} = 0$ yields the solution in the following form:

$$\bar{v}' = \sum\nolimits_{s=1}^{2} \bar{v}'_s(\varphi) = \sum\nolimits_{s=1}^{2} \delta\bar{v}_s\left(\gamma_s\varphi\right)$$

where $\gamma_1 = \sqrt{\bar{\Omega} + \mu^2\bar{\kappa}^2}$, $\gamma_2 = \mu\bar{\kappa}$, and $\delta\bar{v}_s$ = const.

Then, from the equation for perturbation of pressure $\bar{p}'_\varphi = \bar{v}'_{\varphi\varphi} - \bar{v}'\left(\bar{\Omega} + \mu^2\bar{\kappa}^2\right)$ we obtain $\bar{p}' \equiv p'\rho h_\infty^2/\eta^2 = f_0 - \delta\bar{v}_2\bar{\Omega}\gamma_2^{-1}\mathrm{ch}(\gamma_2\varphi)$. For perturbations of the velocity components we have the following non-homogeneous equations: $\bar{u}'_{\varphi\varphi} - \gamma_1^2\bar{u}' = \bar{u}_\varphi\bar{v}'$ and $\bar{w}'_{\varphi\varphi} - \gamma_1^2\bar{w}' = i\gamma_2\bar{p}'$. The general solutions to the homogeneous parts of these equations are $\bar{u}'_0 = (\bar{u}'_0)_1 e^{\gamma_1\varphi} + (\bar{u}'_0)_2 e^{-\gamma_1\varphi}$, $\bar{w}'_0 = (\bar{w}'_0)_1 e^{\gamma_1\varphi} + (\bar{w}'_0)_2 e^{-\gamma_1\varphi}$, $(\bar{u}'_0)_s$ = const, and $(\bar{w}'_0)_s$ = const.

The particular solutions to the non-homogeneous equations can be found by the method of variation of the constants in the following form: $\bar{u}' = \bar{u}_1' e^{\gamma_1 \varphi} + \bar{u}_2' e^{-\gamma_1 \varphi}$ and $\bar{w}' = \bar{w}_1' e^{\gamma_1 \varphi} + \bar{w}_2' e^{-\gamma_1 \varphi}$, where functions $\bar{u}_s'(\varphi)$ and $\bar{w}_s'(\varphi)$ satisfy the equations

$$\left\{\begin{array}{l}(\bar{u}_1')_\varphi e^{\gamma_1\varphi} + (\bar{u}_2')_\varphi e^{-\gamma_1\varphi} = 0 \\ (\bar{u}_1')_\varphi (e^{\gamma_1\varphi})_\varphi + (\bar{u}_2')_\varphi \left(e^{-\gamma_1\varphi}\right)_\varphi = \bar{u}_\varphi \bar{v}'\end{array}\right\} \Rightarrow \bar{u}_{1,2}' = f_{1,2} \pm \frac{1}{2\gamma_1}\int e^{\mp\gamma_1\varphi}\bar{u}_\varphi \bar{v}' d\varphi$$

$$\left\{\begin{array}{l}(\bar{w}_1')_\varphi e^{\gamma_1\varphi} + (\bar{w}_2')_\varphi e^{-\gamma_1\varphi} = 0 \\ (\bar{w}_1')_\varphi (e^{\gamma_1\varphi})_\varphi + (\bar{w}_2')_\varphi \left(e^{-\gamma_1\varphi}\right)_\varphi = i\gamma_2 \bar{p}'\end{array}\right\} \Rightarrow \bar{w}_{1,2}' = f_{3,4} \pm \frac{1}{2\gamma_1}\int e^{\mp\gamma_1\varphi} i\gamma_2 \bar{p}' d\varphi$$

After integration we obtain

$$\begin{aligned}\bar{u}' &= f_1 e^{\gamma_1\varphi} + f_2 e^{-\gamma_1\varphi} + \delta\bar{v}_1 \mathrm{sh}(\gamma_1\varphi)\left(\frac{\bar{u}\coth(\gamma_1\varphi)}{2\gamma_1} - \frac{\bar{u}_\varphi}{4\gamma_1^2} + \frac{\bar{u}_{\varphi\varphi}\coth(\gamma_1\varphi)}{8\gamma_1^3}\right) \\ &- \delta\bar{v}_2 \mathrm{sh}(\gamma_2\varphi)\left(\frac{\bar{u}_\varphi}{\bar{\Omega}} + \frac{2\gamma_2\bar{u}_{\varphi\varphi}\coth(\gamma_2\varphi)}{\bar{\Omega}^2}\right) + f_5\end{aligned}$$

$$\frac{\bar{w}'}{i\gamma_2} = f_3 e^{\gamma_1\varphi} + f_4 e^{-\gamma_1\varphi} + \frac{\delta\bar{v}_2}{\gamma_2}\coth(\gamma_2\varphi) - \frac{f_0}{\gamma_1^2} + f_6.$$

By substitution of the solutions into the Navier-Stokes equations, it is found that $f_{5,6} = 0$. From the equation of continuity it follows that $\bar{w}' = i\gamma_1\gamma_2^{-1}\delta\bar{v}_1 \mathrm{ch}(\gamma_1\varphi) + i\delta\bar{v}_2 \mathrm{ch}(\gamma_2\varphi)$; therefore, $f_0\gamma_1^{-2} = f_6 = 0$ and $f_{3,4} = \gamma_1\delta\bar{v}_1/2\gamma_2^2$. The relation between f_1 and f_2 can be derived from the condition $\bar{u}'|_{\varphi=0} = 0$: $f_2 = -f_1 - \bar{u}_{\varphi\varphi}\delta\bar{v}_1/8\gamma_1^3 + 2\gamma_2\bar{u}_{\varphi\varphi}\delta\bar{v}_2/\bar{\Omega}^2$.

If $\mathrm{Re}\cdot\mathrm{Pr} \sim O\left(\varepsilon^{-1}\right) \gg 1$, then the heat transfer equation yields $\sigma_t' = -(\sigma_x)_{\mathrm{cr}}\, u'$ (at y = h). Here, we substitute the obtained solution for $\bar{u}'$ and use the third boundary condition at the perturbed free surface $y = h + h'$ (Sharypov and Medvedko, 2000):

$$\begin{aligned} v' &= h_t' \\ u_y' + h'\, u_{yy}|_{y=h} + v_x' &= \sigma_x'\eta^{-1} \\ p' + (h_{xx}' + h_{zz}')\,\sigma + h_x'\eta u_y|_{y=h} &= 2\eta v_y' + h'\rho\,|\mathbf{g}|\cos\theta \\ w_y' + v_z' &= \sigma_z'\eta^{-1}\end{aligned} \tag{5}$$

This allows us to find f_1:

$$\begin{aligned}2f_1 \mathrm{sh}\left(\gamma_1\bar{h}\right) &= \delta\bar{v}_1 \mathrm{sh}\left(\gamma_1\bar{h}\right)\left[-2\bar{\Omega}B^{-1}\left(1+\bar{\Omega}/2\gamma_2^2\right) - \coth\left(\gamma_1\bar{h}\right)\bar{u}|_{\varphi=\bar{h}}/2\gamma_1 + B/4\gamma_1^2 - \bar{u}_{\varphi\varphi}/8\gamma_1^3\right] \\ &+ \delta\bar{v}_2 \mathrm{sh}\left(\gamma_2\bar{h}\right)\left\{\bar{\Omega}^{-1}\,\bar{u}|_{\varphi=\bar{h}} - 2\bar{\Omega}B^{-1} + 2\bar{\Omega}^{-2}\gamma_2\bar{u}_{\varphi\varphi}\left[\mathrm{ch}\left(\gamma_2\bar{h}\right) - \exp\left(-\gamma_1\bar{h}\right)\right]/\mathrm{sh}\left(\gamma_2\bar{h}\right)\right\}\end{aligned}$$

where Re = $3\mathrm{Re}_\Gamma$; $B \equiv \mathrm{Re}\bar{\sigma}_x = \bar{u}_\varphi|_{\varphi=\bar{h}}$; $\bar{\sigma}_x = (\sigma_x)_{\mathrm{cr}}\left(\rho h_\infty\,|\mathbf{g}|\sin\theta\right)^{-1}$; and $\bar{h} = h/h_\infty$.

Using the first and second boundary conditions [Eq. (5)], we can write: $\left(\bar{u}' + \bar{v}'\bar{u}_{\varphi\varphi}/\bar{\Omega}\right)\big|_{\varphi=\bar{h}} = 0$, since $\lambda = 0$. Substituting the solutions here, we obtain

$$\begin{aligned}0 &= \delta\bar{v}_1 \mathrm{sh}\left(\gamma_1\bar{h}\right)\left\{0.5\left[1-\coth^2\left(\gamma_1\bar{h}\right)\right]\bar{u}|_{\varphi=\bar{h}} + \coth\left(\gamma_1\bar{h}\right)B/2\gamma_1\right. \\ &\left. + \bar{\Omega}^{-1}\bar{u}_{\varphi\varphi}\left(1-\bar{\Omega}/4\gamma_1^2\right) - 2\gamma_1 B^{-1}\bar{\Omega}\left(1+\bar{\Omega}/2\gamma_2^2\right)\coth\left(\gamma_1\bar{h}\right)\right\} \\ &+ \delta\bar{v}_2 \mathrm{sh}\left(\gamma_2\bar{h}\right)\left(\bar{\Omega}^{-1}\left[\gamma_1\coth\left(\gamma_1\bar{h}\right) - \gamma_2\coth\left(\gamma_2\bar{h}\right)\right]B - 2\bar{\Omega}^{-2}\gamma_2^2\bar{u}_{\varphi\varphi} - 2\gamma_1 B^{-1}\bar{\Omega}\coth\left(\gamma_1\bar{h}\right)\right. \\ &\left. + 2\bar{\Omega}^{-2}\gamma_1\gamma_2\bar{u}_{\varphi\varphi}\coth\left(\gamma_1\bar{h}\right)\left\{\mathrm{ch}\left(\gamma_2\bar{h}\right) - \exp\left(-\gamma_1\bar{h}\right)\left[1+\mathrm{th}\left(\gamma_1\bar{h}\right)\right]\right\}/\mathrm{sh}\left(\gamma_2\bar{h}\right)\right)\end{aligned}$$

If $\left|\gamma_s\bar{h}\right| \ll 1$, then the exponents represented by the first terms of the expansion series and the last equation can be approximately written as follows:

$$\frac{\delta\bar{v}_1 \mathrm{sh}\left(\gamma_1\bar{h}\right)}{\delta\bar{v}_2 \mathrm{sh}\left(\gamma_2\bar{h}\right)}\left\{1 + \frac{\bar{\Omega}}{2\gamma_2^2} - \frac{B\bar{h}}{2\bar{\Omega}}\left[\frac{\bar{u}|_{\varphi=\bar{h}}}{2}\left(1 - \frac{1}{\gamma_1^2\bar{h}^2}\right) + \frac{B}{2\gamma_1^2\bar{h}} + \frac{\bar{u}_{\varphi\varphi}}{\bar{\Omega}}\left(\frac{3}{4} + \frac{\gamma_2^2}{4\gamma_1^2}\right)\right]\right\} = \frac{B\bar{h}\bar{u}_{\varphi\varphi}}{\bar{\Omega}^2} - 1$$

For the critical regime [when $\bar{u}\left(0,\bar{h}\right)$ = 0], $\bar{u}_{\varphi\varphi}\bar{h}/2$ = B. Thus, the derived equation has a more compact form:

$$\delta\bar{v}_1\text{sh}\left(\gamma_1\bar{h}\right)\left[1+\bar{\Omega}/2\gamma_2^2-B^2\bar{\Omega}^{-2}\right]+\delta\bar{v}_2\text{sh}\left(\gamma 2\bar{h}\right)\left[1-2B^2\bar{\Omega}^{-2}\right]=0. \tag{6}$$

The third condition in Eq. (5), $\left(\bar{p}'-2\bar{v}'_{\varphi}-\bar{v}'A/\bar{\Omega}\right)\big|_{\varphi=\bar{h}}=0$, yields the following relation between the amplitudes of the perturbation wave modes:

$$\delta\bar{v}_1\text{sh}\left(\gamma_1\bar{h}\right)\left(1+A\bar{h}/2\bar{\Omega}\right)=-\delta\bar{v}_2\text{sh}\left(\gamma_2\bar{h}\right)\left[1+\bar{\Omega}/2\gamma_2^2+A\bar{h}/2\bar{\Omega}\right] \tag{7}$$

where $A=\left(\mu^2\bar{\kappa}^2\bar{\sigma}+\cot\theta\right)Re$; $\bar{\sigma}=\sigma\left(\rho h_\infty^2\,|\mathbf{g}|\sin\theta\right)^{-1}=l_\sigma^2\left(h_\infty^2\sin\theta\right)^{-1}$; and l_σ is the capillary constant of the liquid. Compatibility between Eqs. (6) and (7) determines the dispersion relation $\Omega(\kappa)$.

If local heating is absent ($\bar{\sigma}_x=0$), then these equations describe the well-known case of capillary-gravitational waves propagating along the surface of a viscous liquid (Landau and Lifshits, 1987):

$$2\gamma_2^2\left[\text{coth}\left(\gamma_1\bar{h}\right)\gamma_1/\gamma_2-\text{coth}\left(\gamma_2\bar{h}\right)\right]=\bar{\Omega}\text{coth}\left(\gamma_2\bar{h}\right)\left(2+\bar{\Omega}/2\gamma_2^2\right)+A/2\gamma_2$$

Since $\left|\gamma_{1,2}\bar{h}\right|\ll 1$, the result is $\bar{\Omega}^2+4\gamma_2^2\bar{\Omega}+A\bar{h}\gamma_2^2=0$. The roots describe perturbations propagating in the z-direction with a certain phase velocity and damping due to viscous dissipation:

$$\Omega=-2\kappa^2\eta/\rho\pm i\sqrt{\kappa^2 h\,|\mathbf{g}|\cos\theta+\kappa^4\sigma h/\rho-\left(2\kappa^2\eta/\rho\right)^2}$$

Taking into account the local heating of the gravity-driven thin liquid film ($\bar{\sigma}_x\neq 0$), we derive the generalized dispersion relation ($\kappa h_\infty\ll 1$):

$$\bar{\Omega}^2\left(\bar{\Omega}^3-2\kappa^2h_\infty^2B^2+4\kappa^2h_\infty^2\bar{\Omega}^2\right)+A\bar{h}\kappa^2h_\infty^2\left(\bar{\Omega}^3+2\kappa^2h_\infty^2B^2\right)+4\kappa^4h_\infty^4B^2\bar{\Omega}=0 \tag{8}$$

Since $\bar{\Omega}\equiv\bar{\Omega}_r+i\bar{\Omega}_{\text{im}}$, we can write the equation for the imaginary part of the increment as follows:

$$\bar{\Omega}_{\text{im}}^5-\bar{\Omega}_{\text{im}}^3A\bar{h}\gamma_2^2-\bar{\Omega}_{\text{im}}^2 10\bar{\Omega}_r^3+\bar{\Omega}_{\text{im}}\left(5\bar{\Omega}_r^4+3\bar{\Omega}_r^2A\bar{h}\gamma_2^2-4\bar{\Omega}_r\gamma_2^2B\right)=0$$

where one of the roots is $\bar{\Omega}_{\text{im}}$ = 0. This means that the absence of phase velocity in the perturbations is in the z-direction. Precisely this type of solution corresponds to the 3D flow regime observed in the experiments. Thus, we assume $\bar{\Omega}_{\text{im}}$ = 0 in Eq. (8).

Let us consider the limit of long-wavelength perturbations: $\kappa h_\infty\to 0$. Then, the first approximation is Ω_1^3 = $2\kappa^2\left(\sigma_x\right)_{\text{cr}}^2/\rho\eta\geq 0$, and these perturbations are unstable due to thermocapillarity. The stabilizing effects of viscosity, surface tension, and hydrostatics are insignificant in the long wave part of the spectrum. The relative orders of magnitude of the terms neglected in Eq. (8) are small: Ω = $\Omega_1\left[1+\text{O}\left(\kappa^{2/3}h_\infty^{2/3}\right)\right]\approx\Omega_1$, since $\kappa h_\infty\ll 1$.

Substituting the second approximation, $\Omega\approx\Omega_2=\Omega_1+\Omega'$, $|\Omega_1|\gg|\Omega'|$, into Eq. (8), we neglect the terms $(\Omega')^m$, $m>1$, and obtain: $\bar{\Omega}'=-2\kappa^2h_\infty^2-2A\bar{h}\kappa^2h_\infty^2/3\bar{\Omega}_1$, thus

$$\Omega\approx\Omega_1-2\kappa^2\eta/\rho-2h\,|\mathbf{g}|\,\kappa^2\left(\kappa^2l_\sigma^2+\cos\theta\right)/3\Omega_1 \tag{9}$$

Although the approximate dispersion relation (9) does not describe the dependence for short-wavelength perturbations, it is non-monotonic and allows us to find the period of linear perturbation Λ_* with the highest amplification rate. This estimation will be acceptable if $2\pi h_\infty/\Lambda_*\ll 1$. We analyze dependency (9) assuming $\kappa^2l_\sigma^2\gg\cos\theta$ and neglecting the effect of viscous dissipation. Then, the condition $d\Omega/d\kappa=0$ gives

$$\left(\Lambda_*/2\pi\right)^8\approx 250\left(h/3\right)^3l_\sigma^6\left(\text{Re}\bar{\sigma}_x^4h_\infty\sin^3\theta\right)^{-1} \tag{10}$$

According to expression (10): $\Lambda_*\sim\left(\sin\theta\right)^{-q}$. When Re_Γ = const, the thickness depends on the angle, $h\sim h_\infty\sim\left(\sin\theta\right)^{-1/3}$, and the exponent, q = 11/24 ≈ 0.46. The value obtained in experiments is q_{exp} ≈ 0.5 (Kabov, 1999, 2010;

Kabov et al., 1999). Thus, the results of the linear analysis of stability satisfactorily reproduce the dependence of the rivulet flow period on the inclination angle.

From the numerical modeling (Kabov, 2010; Frank and Kabov, 2006) the period of 3D flow structure is found to be proportional to $\sigma^{0.365}$. The obtained result [Eq. (10)] predicts the dependence: $\Lambda_* \sim \left(l_\sigma^6\right)^{1/8} \sim \sigma^{3/8} = \sigma^{0.375}$, which corresponds to these calculations better than the proportionality $\sigma^{1/3}$ from Tiwari and Davis (2009).

Expression (10) does not provide explicit dependence $\Lambda_*\left(\mathrm{Re}_\Gamma\right)$ because many of the parameters are connected to Re_Γ in the critical regime, i.e., the heat flux, surface tension, etc. To analyze $\Lambda_*\left(\mathrm{Re}_\Gamma\right)$ it is necessary to note the dependences of these parameters on Re_Γ in the critical regime (for example, from the 2D steady-state numerical solutions). We can also compare the absolute value of Λ_* determined from Eq. (10) with the experimental data (Kabov, 1999) on the period of observed rivulet flow in 25% C_2H_5OH + 75% H_2O: $\Lambda_{\text{exp}} \approx 3.26\,\mathrm{Re}_\Gamma^{1/6} l_\sigma \approx 7 \times 10^{-3}$ m. Equation (9) with the same parameters Re_Γ, θ, T_∞, h_∞, $h(0)$, σ_x, and l_σ as previously mentioned for 2D modeling gives $\Lambda_* \approx 7.7$ mm (the corresponding wave number: $\kappa_* h_\infty = 2\pi h_\infty/\Lambda_* \approx 0.1 \ll 1$). This result is also in good quantitative agreement with the experiments.

4. NONLINEAR MODEL AND NUMERICAL SIMULATION

The obtained dispersion relation [Eq. (9)] can be used in the numerical simulation of the dynamics and structure of the 2D film surface in the quasi-linear approximation. First, we write the solutions to the linear problem in following explicit form (taking into account the complex conjugate parts):

$$\bar{h}' = \sum_{j=1}^{\infty}\left[\left(\delta\bar{h}_1\right)_j + \left(\delta\bar{h}_2\right)_j\right]\exp\left(\Omega_j t\right)\cos\left(\kappa_j z\right)$$

From Eqs. (3) and (7) and the first equation in Eq. (5) were obtain

$$\bar{v}'\left(\varphi\right)\big|_{\varphi=\bar{h}} = \sum_{j=1}^{\infty}\bar{\Omega}_j\delta\bar{h};\quad \frac{\delta\bar{h}_1}{\delta\bar{h}_2} = \left.\frac{\bar{v}_1'(\varphi)}{\bar{v}_2'(\varphi)}\right|_{\varphi=\bar{h}} = \frac{\delta\bar{v}_1\mathrm{sh}\left(\gamma_1\bar{h}\right)}{\delta\bar{v}_2\mathrm{sh}\left(\gamma_2\bar{h}\right)} = -1 - \left(\frac{2\gamma_2^2}{\bar{\Omega}} + \frac{A\bar{h}\gamma_2^2}{\bar{\Omega}^2}\right)^{-1}$$

i.e., the total amplitude is $\delta\bar{h} \equiv \delta\bar{h}_1 + \delta\bar{h}_2 = -\delta\bar{h}_2\left(2\gamma_2^2/\bar{\Omega} + A\bar{h}\gamma_2^2/\bar{\Omega}^2\right)^{-1}$. This variable is the unique unknown parameter:

$$\bar{v}_2'\left(\varphi\right)\big|_{\varphi=\bar{h}} = \delta\bar{v}_2\mathrm{sh}\left(\gamma_2\bar{h}\right) = \bar{\Omega}\delta\bar{h}_2 = -\delta\bar{h}\left(2\gamma_2^2 + A\bar{h}\gamma_2^2/\bar{\Omega}\right)$$

$$\bar{w}'\left(\varphi\right)\big|_{\varphi=\bar{h}} = i\left[\frac{\gamma_1\delta\bar{v}_1\mathrm{sh}\left(\gamma_1\bar{h}\right)}{\gamma_2\delta\bar{v}_2\mathrm{sh}\left(\gamma_2\bar{h}\right)}\coth\left(\gamma_1\bar{h}\right) + \coth\left(\gamma_2\bar{h}\right)\right]\cdot\bar{v}_2'\left(\varphi\right)\big|_{\varphi=\bar{h}} \approx \frac{i\bar{\Omega}}{\gamma_2\bar{h}}\delta\bar{h}$$

or, with the complex conjugate part, $\bar{w}'\big|_{\varphi=\bar{h}} \approx -\delta\bar{h}\bar{\Omega}_1/\gamma_2\bar{h}\exp\left(\Omega t\right)\sin\left(\kappa z\right)$, here it is $\coth\left(\gamma_s\bar{h}\right) \approx \left(\gamma_s\bar{h}\right)^{-1}$. Analogously, we can write the following equations for other unstable disturbances:

$$\bar{p}'\left(\varphi\right)\big|_{\varphi=\bar{h}} = -\bar{\Omega}\coth\left(\gamma_2\bar{h}\right)/\gamma_2\cdot\bar{v}_2'(\varphi)\big|_{\varphi=\bar{h}} \approx \delta\bar{h}\left(2\bar{\Omega}/\bar{h}\bar{h} + A\right)$$

$$\delta\bar{\sigma} = \mathrm{Re}^{-1}\left.\left(\bar{v}' + \frac{\bar{w}_\varphi'}{i\gamma_2}\right)\right|_{\varphi=\bar{h}} = \frac{\delta\bar{h}}{\mathrm{Re}}\left(\frac{\bar{\Omega}^2}{\gamma_2^2} + 4\bar{\Omega} + A\bar{h}\right) \approx \frac{\delta\bar{h}\bar{\Omega}_1^2}{\mathrm{Re}\gamma_2^2} = \delta\bar{h}\left(\frac{4\mathrm{Re}\bar{\sigma}_x^4}{\gamma_2^2}\right)^{1/3}$$

$$\bar{u}'(\varphi)\big|_{\varphi=\bar{h}} = -\delta\bar{\sigma}\bar{\Omega}/\bar{\sigma}_x \approx -\delta\bar{h}\bar{\Omega}_1^3 B^{-1}\gamma_2^{-2} = -2B\delta\bar{h}$$

Thus, we determine the amplitudes of all of the perturbed parameters in the vicinity of the free surface. Analysis of phase relations of the unstable harmonics reveals the following. The phases of the pressure, surface tension, and x-velocity disturbances coincide with the perturbation phase of the surface coordinate, which means the temperature at the free surface decreases if the film thickness increases. In the hollows the temperature is higher, and the x-component of the velocity is directed opposite to the undisturbed flow. The total pressure grows by thickening of the layer, capillary forces, and hydrostatic pressure. The transverse component of the velocity is directed from the lowest part of the surface toward the apexes of the crests, gathering the liquid from the hollows into periodic rivulets.

The kinematic condition at the free surface (without the stationary part) is obtained as follows:

$$h'_t = v' - u'\left(h + h'\right)_x - uh'_x - w'h'_z, \quad y = h$$

where the perturbations of the velocity components can be expressed through h' from the linear analysis. Thus, we only take into account quadratic nonlinearity (the quasi-linear approach). Due to $\lambda = 0$, $h\left(t, x, z\right) = h\left(x\right) + h'\left(t, z\right)$, and the evolution equation for the coordinate of the unstable free surface takes the following form:

$$h'_t = v' - u'h_x - w'h'_z, \quad |x| \le \mu L \tag{11}$$

where $v'|_{y=h} = \Omega h'$; $w'|_{y=h} = h^{-1}\left[2\left(\sigma_x\right)^2_{\text{cr}}/\kappa^4\rho\eta\right]^{1/3} h'_z$; and $u'|_{y=h} = -2\left(\sigma_x\right)_{\text{cr}}\eta^{-1}h'$. All the terms on the right-hand side of Eq. (11) have approximately equal orders of magnitude if $h' \sim h_\infty$, i.e., the nonlinear effects are able to stabilize the amplification of long-wavelength perturbations.

Equation (11) was solved numerically in the region $x \in [-L_-; L_+]$. Outside this region, it was assumed that $\sigma_x = 0$, and the perturbations of the 2D steady-state solution were transferred by the flow downstream. The periodic boundary conditions were set at $z = z_{\text{min}}$ and $z = z_{\text{max}}$ ($z_{\text{min}} - z_{\text{max}} \gg \Lambda_*$). For the numerical simulation we can write $dh/dt \equiv \left(dh'/dt\right)_1 + \left(dh'/dt\right)_2$. At each time step the equation $\left(dh'/dt\right)_1 = \Omega h'$ was solved using the fast Fourier transform (spectral) method, and the second equation $\left(dh'/dt\right)_2 = 2\left(\sigma_x\right)_{\text{cr}} h_x\eta^{-1}h' - \left[2\left(\sigma_x\right)^2_{\text{cr}}/\kappa^4\rho\eta\right]^{1/3} h^{-1}\left(h'_z\right)^2$ was solved using the finite-difference method. The initial condition was the solution $h(x)$ to the 2D steady-state problem with superimposed harmonic perturbations using all possible periods and random small amplitudes, $\pm$ (2–12) $\times 10^{-5}$ mm. The values of the physical parameters were the same as previously given and correspond to the experimental conditions (Kabov, 1999). The simulation results for the surface deformation showed rivulet flow with a spatial period close to Λ_* (see Fig. 2). The established thickness changed periodically from (0.3–0.5)h_∞ up to $2h_\infty$.

With the linear relations between h' and the perturbations of the other parameters we can approximately calculate the distributions of the temperature and velocity at the perturbed free surface. The distributions $T\left(x, h, z\right)$ and $|\mathbf{g}|$ are plotted in Figs. 3 and 4. The difference in the temperatures in the hollows and rivulets reaches 10 K. The ex-

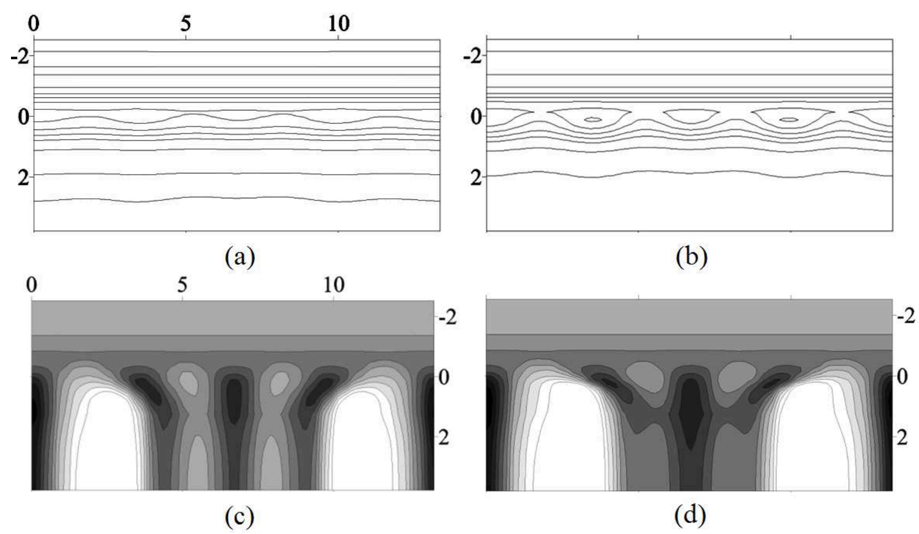

FIG. 2.

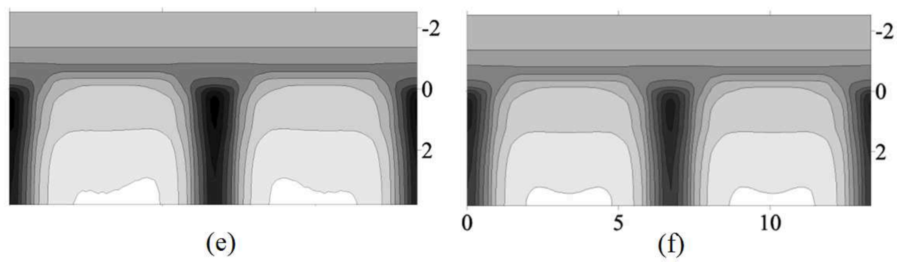

FIG. 2: Development of the periodic structure of the film surface: (a) t = 15 ms; (b) t = 22 ms; (c) t = 117 ms; (d) t = 124 ms; (e) t = 204 ms; (f) t = 584 ms

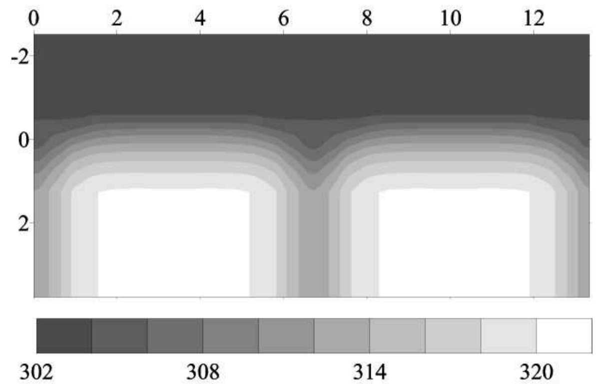

FIG. 3: Steady-state distribution of the temperature at the free surface (K)

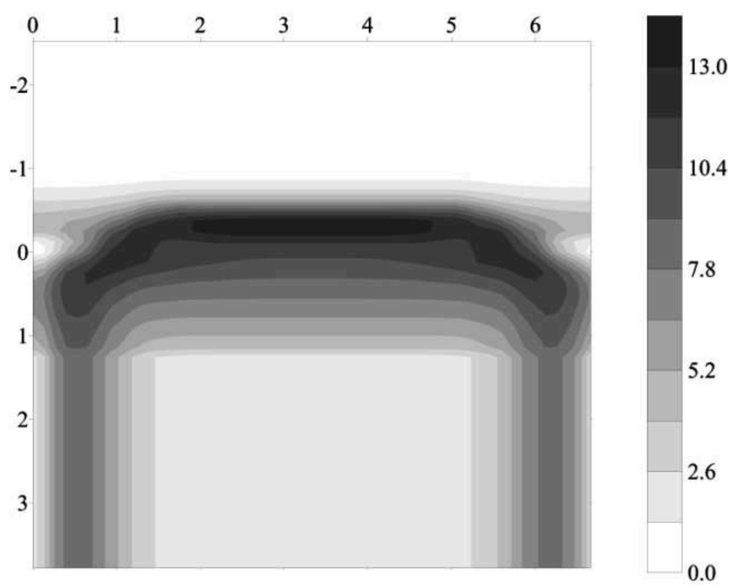

FIG. 4: Steady-state distribution of $|\mathbf{grad}T|$ at the free surface (K/mm)

tremely high temperature gradient (up to 13 K/mm) occurs at the upper edge of the hollows. These values agree with measurements carried out using infrared thermography (Kabov, 1999; Marchuk, 2000).

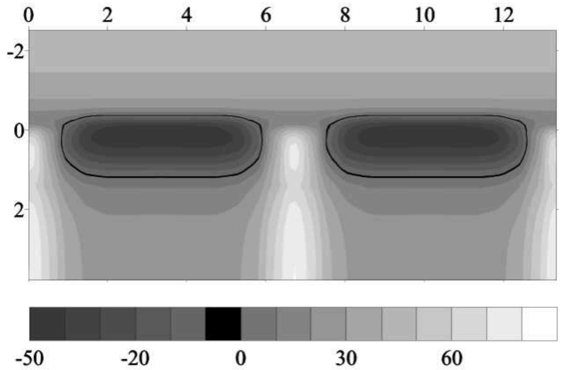

FIG. 5: Steady-state distribution of the x-component of the velocity at the free surface (mm/s)

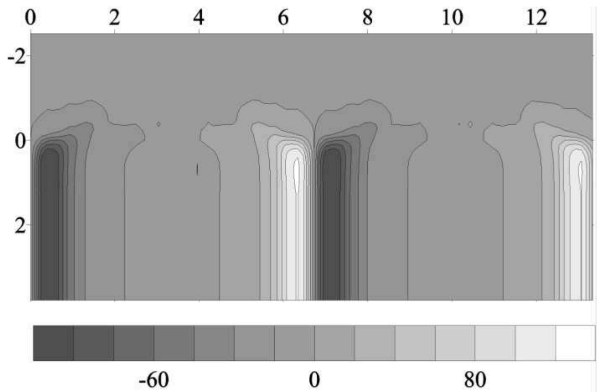

FIG. 6: Steady-state distribution of the z-component of the velocity at the free surface (mm/s)

The distribution of the x-component of the velocity at the free surface is shown in Fig. 5. Zones with negative velocity component values appear at the upper parts of the hollows, indicating reverse flow due to the thermocapillary effect. The velocity of the liquid in the rivulets is about twice the maximal velocity in the isothermal film flow. High local values are also characteristic for the z-component of the velocity at the sides of the rivulets (up to 100 mm/s), see Fig. 6.

5. CONCLUSIONS

The results of the linear analysis of the stability of the 2D critical flow regime of a gravity-driven thin liquid film with local heating demonstrates long-wavelength instability in this regime and predicts the dependence of the characteristic period of the 3D flow structure on the main physical parameters. The obtained analytical results are in good agreement with the known experimental data. The numerical simulation, based on the derived quasi-linear evolution equation, allowed us to calculate the steady-state periodic structure of the film surface and the distributions of the temperature and velocity components at the free surface. The results of the simulation reproduced the main quantitative and qualitative aspects of the phenomenon observed in the experiments.

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