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LIMITING STRETCHES OF LIQUIDS AT A PULSED FLOW ON A SHARP EDGE

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A method of investigating the fluctuation formation of cavitation bubbles at a pulsed flow of the sharp edge of a solid inclusion has been developed and tested. On a sharp edge the flow rate increases locally and according to the Bernoulli equation the pressure drops. Under certain conditions the pressure on the edge of a solid inclusion, which flows by a pulsed liquid stream, becomes negative and may reach hundreds of atmospheres. It is shown that as a result of high localization of a liquid stretch the shock regime of cavitation on fluctuation centers is realized in experiments for a short time period in a small volume.

KEY WORDS: *fluctuation nucleation, negative pressure, shock regime, cavitation*

1. INTRODUCTION

It is common knowledge that the regime of homogeneous fluctuation nucleation is most probable during rapid superheating or stretching of small portions of liquids. As a rule, under such conditions completed vaporization centers are not revealed; therefore, the shock regime (SR) of cavitation or boiling takes places (Pavlov, 1988; Skripov et al., 1988; Kuznecthov et al., 2015; Baidakov et al., 2016, 2017). It is well known that on the edges of solid inclusions one can observe the formation of vortices and the separation of a laminar flow. Of particular interest are regimes that are established quickly at a high rate when the laminar character of the flow of the edges is retained for a certain time. In these regimes one can observe a considerable pressure drop on the edges, and as a result the development of cavitation.

The theory of homogeneous fluctuation nucleation is well developed (Skripov, 1974); therefore, calculation of the bubble nucleation rate from a well-known temperature and pressure field presents no difficulty. The use of a shock wave generated by a special device is suggested to create a controllable pulse of the liquid velocity. It is known that the passage of a shock wave is accompanied by a co-current flow, the rate of which depends on the adiabatic compressibility of the medium $[(d\rho/dp)_a]$ and the wave amplitude $(p_2 - p_1)$. To calculate the rate of the co-current flow after a shock in the liquid, the following well-known formula can be used (Landau and Lifshitz, 1986):

$$V = \sqrt{(p_2 - p_1)\left[(1/\rho_1) - (1/\rho_2)\right]} \tag{1}$$

where p_2 and p_1 are the pressure in the shock wave and the initial pressure, respectively; and ρ_2 and ρ_1 are the corresponding liquid densities. Provided that $[(p_2 - p_1)/\rho_1]\,(d\rho/dp)_a \ll 1$, from formula (1) one can obtain the following simple formula for the co-current flow rate:

$$V \approx \sqrt{(d\rho/dp)_a}(p_2 - p_1)/\rho_1 \tag{2}$$

Calculation of a rate and pressure field is difficult because of the complex hydrodynamics close to the solid inclusion.

The agreement between the checked experimental results and homogeneous fluctuation nucleation theory may serve as an indicator of the correctness of a calculation model. Given subsequently are the procedure for calculating the pressure and nucleation rate on an edge, the experimental technique, and the experimental results.

2. PRESSURE ON THE EDGE AT A LAMINAR FLOW OF A PLATE

Let us examine the two-dimensional potential of a laminar liquid flow in a band $-b < y < b$, $-\infty < x < \infty$ with a solid flat thin rectangular inclusion of width $2h \ll 2b$, thickness 2α, and an infinite length situated across the flow (Fig. 1). The inclusion ends with round mouths of radius α ($\alpha \ll h$). Since the problem is symmetrical, it will suffice to consider the region $0 < y < b$.

Let us use the conformal transformation of a band with a cut to another band and obtain the potential far from the inclusion mouth (Koppenfels and Stallmann, 1959; Pavlov, 1979). To calculate the field close to the mouth, we conformally transform a half-plane to a half-plane with a thrown out semicircle. After replacing the complex variable by the square root of the new variable it is possible to join the potential obtained with that in the band with a cut (Koppenfels and Stallmann, 1959). Now the potential close to the mouth of the solid inclusion can be obtained as follows:

$$\psi(r,\varphi) = \sqrt{\frac{4}{\pi}\alpha b \cdot \sin\left(\frac{\pi h}{2b}\right)} \cdot \left[\sqrt{\frac{r}{\alpha}} + \sqrt{\frac{\alpha}{r}}\right] V \cdot \cos\left(\frac{2\varphi+\pi}{4}\right) \tag{3}$$

where V is the flow rate far from the solid inclusion; r is the distance from the mouth center; and the angle φ is reckoned from the direction of the x-axis (the direction of the flow). Using the potential obtained, one can determine the flow rate near the inclusion mouth as follows:

$$v(r,\varphi) = -\frac{d}{rd\varphi}\psi(r,\varphi) = \sqrt{\frac{\alpha b}{\pi} \cdot \sin\left(\frac{\pi h}{2b}\right)} \cdot \left[\sqrt{\frac{r}{\alpha}} + \sqrt{\frac{\alpha}{r}}\right] \frac{V}{r} \cdot \sin\left(\frac{2\varphi+\pi}{4}\right) \tag{4}$$

In the experimental conditions we always have $h \ll b$; therefore, at $\varphi = \pi/2$ formula (4) is simplified:

$$v(r,\pi/2) = \sqrt{\frac{\alpha h}{2}} \cdot \left[\sqrt{\frac{r}{\alpha}} + \sqrt{\frac{\alpha}{r}}\right] \frac{V}{r} \tag{5}$$

In particular, the maximum rate on the edge before the liquid tears off from the rounded area is equal to

$$v(\alpha,\pi/2) = V\sqrt{2h/\alpha} \tag{6}$$

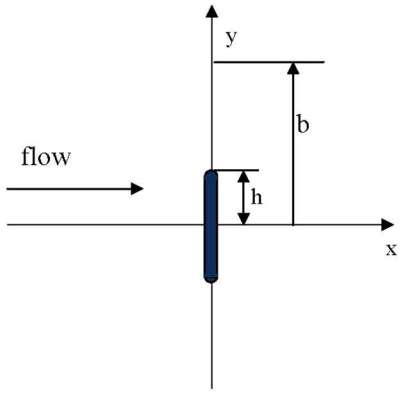

FIG. 1: Simplified scheme of the flow around a solid inclusion

The pressure near the edge is determined from the condition of momentum conservation in an incompressible liquid. At an angle $\varphi = \pi/2$ we have

$$p(r) = p_2 + \left\{1 - \frac{\alpha h}{2}\left[\sqrt{\frac{1}{\alpha r}} + \frac{1}{r}\sqrt{\frac{\alpha}{r}}\right]^2\right\}\rho_2 V^2 \tag{7}$$

On the edge the pressure is equal to

$$p_m \equiv p(r=\alpha) = p_2 - 2\rho_2 V^2 \frac{h}{\alpha}\left(1 - \frac{\alpha}{2h}\right) = p_2 - \left(1 - \frac{\alpha}{2h}\right)p_v; \quad p_v = 2\rho_2 V^2 \frac{h}{\alpha} \tag{8}$$

Figure 2 presents dependences $p_m(p_2)$ in n-hexane for different geometric plate sizes. From Fig. 2 it follows that negative pressures of tens of megapascals may be realized on the edge of the plate at relatively small incident wave amplitude. Furthermore, a linear approximation may be assumed

$$p(r) \approx p_m + \left.\frac{dp}{dr}\right|_{r=\alpha}(r-\alpha) \approx p_m + 2p_v \cdot \frac{r-\alpha}{\alpha} \tag{9}$$

The liquid tensile strength is determined by this formula. It should be mentioned that the calculation ignores the presence on the solid of a boundary layer of thickness $\delta_\nu = K\sqrt{\nu\alpha/V}$, where ν is the coefficient of kinematic viscosity (Schlichting, 1951). The value of the coefficient K in similar problems is usually close to unity but requires refinement from experiments. Under the presented conditions of the experiments $\delta_\nu \ll \alpha$; therefore, the liquid viscosity is subsequently neglected.

3. NUCLEATION RATE ON THE EDGE

Homogeneous fluctuation nucleation theory may be used to calculate the nucleation rate of viable bubbles in a stretched liquid (Skripov, 1974). In experiments, the fluctuation mechanism of bubble nucleation is realized either under very pure conditions or at a sufficiently rapid approach to the spinodal (Pavlov, 2003). The value of the nucleation rate observed is a certain measure of the approach to the spinodal. Its maximum value reaches 10^{37} m^{-3}s^{-1}.

The rate of spontaneous nucleation registered in the experiments considered subsequently is determined from the condition of filling the cylindrical region near the edge with vapor to a radius of 2α. Let us use the well-known approximation for the dependence of the specific fluctuation nucleation rate on the pressure p at a given liquid temperature

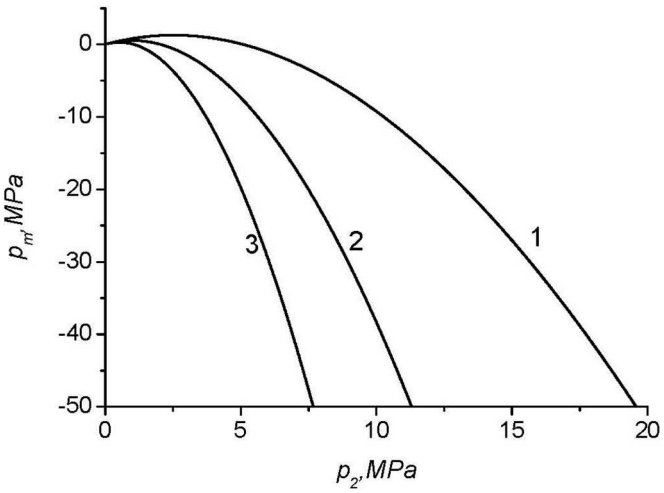

FIG. 2: Dependence of the pressure on the plate edge in n-hexane on the amplitude of an incident wave for plates of different thicknesses (1, 50-μm-thick plate; 2, 20-μm-thick plate; 3, 10-μm-thick plate; plate width, 3 mm)

(Skripov, 1974):

$$J(p) = J_0 \exp\left[-G_P (p - p_0)\right], \quad G_P = -\frac{d}{dp} \ln J(p) \tag{10}$$

Here, the reference value of the rate $J_0 = J(p_0)$, and the coefficient G_p is determined from kinetic nucleation theory (Pavlov, 1988). We introduce the time dependence of the bubble radius in the following power form:

$$R(t) = \beta t^k \tag{11}$$

Equation (11), in general terms, expresses the power law of bubble growth. In the region of negative pressures, it takes the form $R(t) = \sqrt{2(p_0 - p)/3\rho} \cdot t$ (Labuntsov and Yagov, 2000). In this case, $k = 1$ and $\beta = \sqrt{2(p_0 - p)/3\rho}$. The specific volume of the vapor formed during time t under isothermal conditions is evidently equal to

$$\Omega''(t, r) = \frac{4}{3}\pi\beta^3 \int_0^t J\left[p(r)\right] \cdot (t - \tau)^{3k} \, d\tau \tag{12}$$

Here

$$J\left[p(r)\right] = J(p_m) \exp\left[-2p_v G_p \cdot \frac{r - \alpha}{\alpha}\right]$$

The time it takes to establish the pressure of a shock wave in experiments does not exceed 1 μs; therefore, further processes at the front of a shock wave are not considered. We assume that p_m and p_v are time independent. It is assumed that the rate of the co-current flow is formed instantly at the front of the shock wave and remains constant during the time it takes for the wave to pass. Therefore, the pressure in formula (12) is time independent. Also, formula (12) takes into account the fact that owing to the strong pressure dependence of the nucleation rate practically has all of the bubbles grow on the inclusion edge. This gives grounds to calculate coefficient β at pressure p_m on the edge and remove it from the sign of the integral. The volume of the vapor in a cylindrical layer near the edge, for instance, with a radius from α to 2α and a unit length is equal to

$$\Omega(t) = 2\pi \int_\alpha^{2\alpha} \Omega''(t, r) \, r dr \tag{13}$$

The vapor will totally fill this volume if $\Omega(t) = \pi 4\alpha^2 - \pi\alpha^2 = 3\pi\alpha^2$. As a result, after a number of transformations, we obtain the following condition of filling a cylindrical layer near the edge with vapor for time t:

$$J(p_m) = \frac{9(p_v G_p)^2 (3k + 1)}{2\pi^2 \beta^3 t^4 \left[1 - (4p_v G_p + 1)\exp(-2p_v G_p) + 2p_v G_p\right]} \tag{14}$$

Experimentally, the condition $p_v G_p \gg 1$ is fulfilled; therefore, we then use the following formula to calculate the fluctuation nucleation rate registered in the experiment:

$$J(p_m) \approx \frac{9(p_v G_p)(3k + 1)}{4\pi^2 \beta^3 t^4} \tag{15}$$

This is the nucleation rate registered in the experiment for the pressure on the edge, which ensures the formation of a vapor filament on the edge in the pulse time. In experiments, it is proposed to obtain the highest possible nucleation rate; therefore, measurements are conducted on short shock waves for durations of less than 5 μs.

4. MEASURING TECHNIQUE AND RESULTS

The scheme of the operating chamber of the experimental setup is given in Fig. 3. A thin plate, 1.5–3 mm wide (Fig. 3, No. 1), is fixed with the help of a copper holder (Fig. 3, No. 2) at the center of the chamber (Fig. 3, No. 3) at

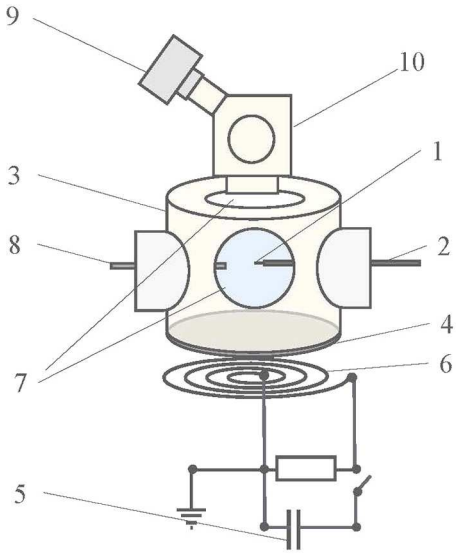

FIG. 3: Scheme of the operating chamber of the setup (1, thin plate; 2, copper holder; 3, chamber body; 4, membrane; 5, high-voltage capacitor; 6, coil; 7, windows; 8, pressure transducer; 9, camera; 10, microscope)

a depth of 5–10 mm from the surface of the liquid filling the chamber. To obtain a shock wave, the technique found in Kedrinskii (2005) and Vinogradov and Pavlov (2016) is used. A pressure wave with a duration of 3–5 μs and a front of ~ 1 μs is created by a membrane (Fig. 3, No. 4), which is the chamber bottom, at a capacitor (Fig. 3, No. 5) discharge on the coil (Fig. 3, No. 6). The shape of the pressure wave is shown in Fig. 4. The shock wave amplitude is controlled by the voltage across the capacitor in the range from 1 to 15 MPa. The setup is preliminarily calibrated. For this purpose, a pulse pressure transducer is installed in the chamber instead of the upper window (Fig. 3, No. 7) at the depth of the plate immersion. The dependence of the pulse pressure amplitude on the voltage across the capacitor is taken. Then, the transducer is removed and the chamber is closed by the upper window. Experiments are conducted in the following way. A microscope (Fig. 3, No. 10) with a camera focused on the plate edge (Fig. 3, No. 9) is fixed over the upper window. Through the side window the plate is illuminated by a pulsed light source with a flash duration of less than 1 μs. The camera is shaded such that no foreign light falls on the object to be photographed. With an open

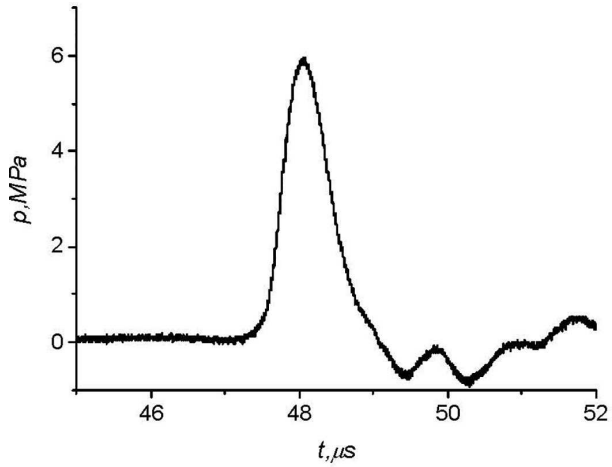

FIG. 4: Oscillogram of a pressure pulse

shutter of the camera the plate is illuminated by the pulsed source when a shock wave passes through it. The response time of the pulsed light source is adjusted by the oscillograms of a light pulse and a piezoelectric ceramic pressure transducer (Fig. 3, No. 8) located in the liquid at the same depth as the plate (Fig. 3, No. 1). The plate is photographed at a constant increase in the shock wave amplitude. As long as there is no cavitation on the edge, the edge of the plate is practically invisible, as shown in Fig. 5(b). Figure 5(a) shows a photograph of the plate under constant illumination. The appearance of cavitation bubbles on the edge of the plate causes a bright glow at the edge, as shown in Fig. 5(d). In such a way we determine p_2. Figure 5(c) shows the appearance of individual centers of cavitation in weak places. The negative pressure on the plate edge that causes cavitation is calculated using tabular data on the elastic properties of the liquid and the value of p_2. The rounding off of radius α was taken to be equal to half the plate thickness. The results of the calculations are given in Table 1. For water, the stretch on a sharp edge registered in the experiments, $p_m = -14.4$ MPa, is practically an order of magnitude lower than the theoretical water strength at room temperature (~ -170 MPa). Stretches close to theoretical values have been obtained in experiments on ethanol and n-hexane.

The uneven filling of a stretched volume with vapor revealed in the photographs may be explained not only by the nonuniformity of the edge but also by the instability of the vaporization process. The fact is that close to the first cavitation bubbles the negative pressure decreases, which is why the nucleation rate also decreases. This fact is disregarded in formula (15). For controlled, additional evaluation of the acting nucleation rate we use the formula $J \sim (\Omega t_1)^{-1}$, where Ω is the volume of a stretched liquid and t_1 is the time of the appearance of the first nucleus. The time of the flow detachment from a solid inclusion is usually close to that of the flow of rounding off the radius on the edge ($t_0 \sim \pi\alpha/V$) and under experimental conditions it is ~ 5 μs (Pavlov, 2003), which makes it possible

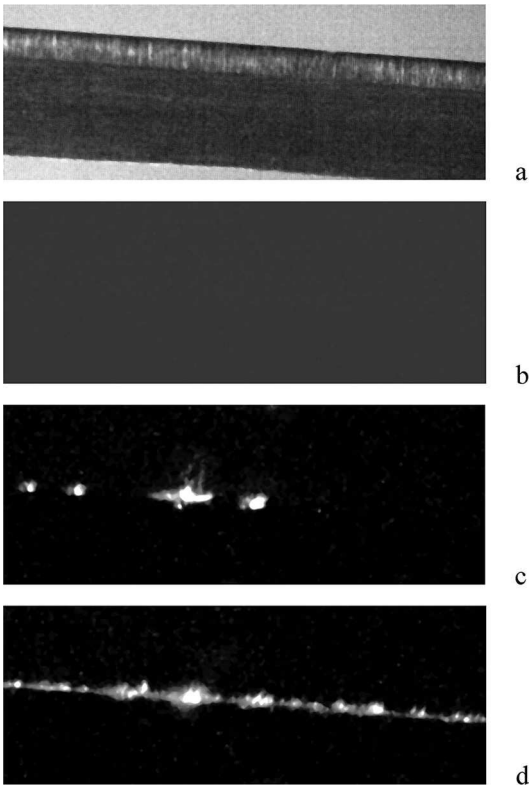

FIG. 5: Photographs of a steel plate in n-hexane with a shock wave passing through it (plate width, 1.5 mm): (a) at a constant atmospheric pressure and constant illumination; (b) amplitude of a shock wave at $p_2 = 6.0$ MPa (the calculated pressure on the edge is $p_m = -11.8$ MPa); (c) $p_2 = 7.0$ MPa and $p_m = -17 \pm 5$ MPa; (d) $p_2 = 8.0$ MPa, $p_m = -23 \pm 5$ MPa, $J(p_m) = 3 \times 10^{19}$ m^{-3}s^{-1}, and $p_{theor} = -23$ MPa. Here, the p_m value was calculated by formula (8), the $J(p_m)$ value was calculated by formula (15), and the p_{theor} values was calculated by formulas of homogeneous nucleation theory (Skripov, 1974)

TABLE 1: Amplitude of a shock wave (p_2), pressure on the edge (p_m), the fluctuation nucleation rate registered in the experiment [$J(p_m)$], and theoretical strength p_{theor} at temperature 25°C

Liquid	Material	p_2 (MPa)	p_m (MPa)	$J(p_m)$ ($m^{-3}s^{-1}$)	p_{theor} (MPa)
Water	Mica ($h = 5$ mm, $\alpha = 10$ μm)	7	−14.4	4×10^{22}	−190
Ethanol	Mica ($h = 5$ mm, $\alpha = 10$ μm)	6	−27	4×10^{19}	−31
n-Hexane	Mica ($h = 3$ mm, $\alpha = 20$ μm)	8.0	−23.0	3×10^{19}	−23.0
	Copper ($h = 3$ mm, $\alpha = 30$ μm)	9.9	−21.7	4×10^{19}	−23.1
	Stainless steel ($h = 3$ mm, $\alpha = 20$ μm)	7.7	−21	4×10^{22}	−25
Dodecane	Stainless steel ($h = 3.5$ mm, $\alpha = 20$ μm)	12.5	−38	4×10^{22}	−42

to additionally evaluate the initial nucleation rate. On the assumption that for the formation of a luminous strip on the edge the creation of one bubble for a length equal to the plate thickness over time ($t_1 = t_0$) will suffice; the nucleation rate in this approximation will be determined by the expression $J \approx (\Omega\pi\alpha/V)^{-1}$. Here, Ω is the volume of the liquid layer near the edge with a length equal to the diameter of a typical registered bubble, where the average pressure within the experimental error is equal to p_m. In the experiments conducted, $\Omega \sim 10^{-17}$ m^3. The nucleation rate calculated this way is 10^{21} to 10^{23} $m^{-3}s^{-1}$, which does not differ greatly from $J(p_m)$ calculated by formula (15). A decrease in rounding off the radius of edge α of less than the thickness of the viscous sublayer $\delta_\nu = K\sqrt{\nu\alpha/V}$ makes no sense; therefore, under the experimental conditions the rate must not exceed V^*, $V^* = K^2\nu/\alpha$, which for the liquids under investigation is fulfilled with a large reserve.

5. CONCLUSIONS

As a result of the theoretical analysis, it is shown that a local decrease in the pressure on a sharp edge when it flows by a current of liquid may serve as a basis for a new method of investigating limiting liquid stretches at nucleation rates in excess of 10^{22} $m^{-3}s^{-1}$. Based on the experimental results, the following conclusions can be drawn:

- To obtain a fast-increasing liquid velocity, it is advisable to use the co-current flow following a shock wave.
- The method has been realized through experiments and tested on several liquids at a temperature of 25°C. The experimental data obtained on limiting stretches of liquids do not contradict the calculations obtained using homogeneous nucleation theory.
- Some difficulties have been revealed in realizing the method. The main difficulty is the preparation and exact determination of the geometric characteristics of a solid inclusion. Also, to calculate the nucleation rate it is important to have a correct evaluation of the time of a liquid stretch (before detachment of the flow) and the thickness of the viscous sublayer.
- It is possible to use the method to obtain information on the limiting strength of various liquids, including chemically unstable liquids. The well-known method of determining the fluctuation nucleation rate in a metastable liquid (Pavlov, 1988) is unsuitable for electrically conducting liquids. This drawback is absent in the method under consideration.
- The investigation of cavitation on a sharp edge is of independent practical importance. The phenomenon being examined is possible in different explosive emergency processes, and therefore is applicable for developing methods of control over explosive vaporization.

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