OPTIMAL DESIGN UNDER UNCERTAINTY OF REINFORCED CONCRETE STRUCTURES USING SYSTEM RELIABILITY APPROACH

Younes Aoues,[1,*] Alaa Chateauneuf,[2] Didier Lemosse,[1] & Abdelkhalak El-Hami[1]

[1] *INSA Rouen, Laboratoire d'Optimisation et Fiabilité en Mécanique des Structures (LOFIMS), France*

[2] *Institut Pascal, Laboratoire de Mécanique et Ingenieries, Université Blaise Pascal, France*

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The design of reinforced concrete (RC) structures involves several kinds of uncertainties, which are usually considered through the partial safety factors prescribed in the codes of practice. The traditional design optimization of RC structures uses deterministic information of the problem. The partial safety factors are used to consider loading fluctuation and the variability of material properties. The use of these safety factors in deterministic optimization usually leads to over-designing structures, as these safety factors are calibrated for a large class of structures. In the deterministic optimization procedure the reliability cannot be controlled. For this reason, the Reliability-Based Design Optimization (RBDO) is devoted to design economical and reliable structures. However, the RBDO problem involves the evaluation of probabilistic constraints performed by the reliability analysis. The most common RBDO formulations are based on a nested optimization problem, with an outer loop for the design optimization and an inner loop for the reliability analysis. Therefore, an expensive computation effort is required to solve the RBDO problem. In this paper, a new RBDO method of RC structures is proposed. The RBDO problem is decomposed to several cycles of deterministic design optimization (DDO) based on new safety factors called Optimal System Safety Factors (OSSF). At each cycle of the DDO, the system reliability analysis is performed to verify the reliability of the optimal design, the OSSF are computed by a probabilistic method on the basis of the previous system reliability analysis, then the updated OSSF are provided for the next cycle of the DDO. The application to the design of an RC structure shows the interest and the efficiency of the proposed method.

KEY WORDS: *Random variables, optimization, system reliability, probability of failure, FORM, RBDO*

1. INTRODUCTION

Structural optimization is frequently applied for effective cost reduction of engineering systems. Traditionally, the deterministic design optimization (DDO) takes into account the uncertainties through the use of partial safety factors defined by standard specifications. The safety factors given in recent codes of practice, such as Eurocodes 2, are calibrated on the basis of a semi-probabilistic approach for the whole domain of application, leading in many cases to over-designed structures. The design of RC structures is based on the verification of several rules defined by the design codes, for instance by the Eurocode [1]. The analysis of RC structures involves several uncertainties related to concrete and steel properties, structural dimensions, and position of steel reinforcement. The fluctuation of loads and the uncertainties regarding the analysis models all contribute to make the performance of the optimal design different from the expected one. Furthermore, the optimization process has a large effect on structural safety. The use

[*]Correspond to Younes Aoues, E-mail: younes.aoues@insa-rouen.fr

of prescribed partial safety factors in designing RC structures, although convenient, is not adapted to determine the reliable design, as these factors are calibrated for a large class of structures [2]. In fact, the partial safety factors are not directly linked to uncertainties of the overall system performance, therefore these safety factors can not guarantee the required safety level of the optimum deterministic design.

Several works have developed various methodologies for optimizing the RC structures using the deterministic optimization, where the structural cost or weight is minimized subjected to the structural constraints specified in the design codes [3]. Many authors are focused to develop efficient optimization method to the RC optimal design, by using a genetic algorithm [4] or a gradient-based algorithm [5]. However, the rational approach consists in finding the best compromise between the cost reduction and the safety assurance. The structural reliability theory provides an appropriate approach to take into account the uncertainties [6]. As safety and economy are the main issues in the design of RC structures, Reliability-Based Design Optimization (RBDO) allows us to reach effectively balanced cost-safety configurations. A practical formulation of the RBDO consists in minimizing the expected cost under probabilistic constraints. Nevertheless, for complex structures the evaluation of probabilistic constraints is computationally expensive, due to the necessity to apply either iterative numerical procedures or Monte Carlo simulations. The RBDO formulation is basically a nested problem, where the outer loop is an optimization procedure for searching the optimal parameters and the inner loop is a second optimization procedure for the evaluation of the reliability constraints [7].

There is another difficulty in the RBDO approach related to the overall system performance. In fact, the RBDO formulation can be devoted either to component reliability (i.e. single limit state function) or to system reliability (i.e. combination of several limit states). Classically speaking, the RBDO is mainly seen as a component reliability problem, and most of works have been carried out in this field [8]. However, the real benefit of the RBDO lies in the balanced choice of target safety with respect to the system failure. To some extent, the system reliability may be considered through the extrapolation of component reliability concepts [9]. Several authors have developed many methodologies for integrating the system reliability in the RBDO. Feng et al. [10] have considered the system reliability as the single reliability constraint. Other authors replace the system reliability by a reliability index of the dominant failure modes [11]. Different numerical applications have addressed simultaneously the system reliability and component reliability [12]. Nevertheless, a direct integration of the system safety in the reliability constraints complicates the RBDO problem and makes it inefficient [13].

In this work, a new methodology of RBDO of RC structures is developed, where the system performance is integrated implicitly in the optimization process. The proposed method is based on the concept of decoupled approach [14], where the RBDO problem is converted into several cycles of the deterministic design based on the safety factor concept. These safety factors are computed on the basis of the system failure. In other words, the new safety factors called Optimal System Safety Factors (OSSF) are calibrated in way that the system reliability requirement is guaranteed. In this way, the system reliability analysis is separated from the optimization procedure, allowing application of classical Deterministic Design Optimization (DDO). At the end of each cycle of DDO (i.e., convergence of the DDO for a given set of OSSF), the OSSF are updated on the basis of the system failure probability, by searching for the optimal component target reliabilities [13]. This OSSF calibration procedure allows us to link the target reliability for the pseudo-deterministic optimization, which are then provided for the next DDO cycle. This scheme is repeated until convergence to minimal cost and target system reliability. The numerical example on RC structure shows the efficiency and the good standing of the proposed method, through comparison with DDO and classical RBDO.

2. DETERMINISTIC DESIGN OPTIMIZATION

The real benefit of design optimization is cost reduction and effective use of structural capacity. Various works on structural optimization deal with the minimization of the construction cost of RC structures. The general formula of the cost function C_c can be written as [15]

$$C_c = C_{cc} + C_{rs} + C_{ss} + C_{fw} \tag{1}$$

where C_{cc} is the concrete cost, C_{rs} is the reinforcement steel cost, C_{ss} is the shear steel cost, and C_{fw} is the formwork cost. The general formulation of the deterministic design optimization is expressed as

$$\begin{aligned} \min_{\mathbf{d}} \quad &: \quad C_c(\mathbf{d}) \\ \text{subject to} \quad &: \quad \begin{cases} G_i(\mathbf{d}, \mathbf{x}_k, \gamma) \leq 0 & i = 1, \ldots, m \\ h_j(\mathbf{d}) \leq 0 & j = m+1, \ldots, n \end{cases} \end{aligned} \tag{2}$$

where $\mathbf{d}$ is the vector of design variables, $\mathbf{x}_k$ is the vector of characteristic values of load actions and material properties (which are not considered as design variables), γ is the vector of partial safety factors, G_i are respectively the ultimate and/or serviceability limit state functions, and h_j are the feasibility constraints (e.g. upper and lower bounds of design variables). In this formulation, the partial safety factors introduced in the deterministic design are assumed to take account of the uncertainties related to resistance and loading, as well as workmanship and construction technology.

3. PROBABILISTIC DESIGN OPTIMIZATION

The challenge in optimal design consists in improving the performance, by reducing the total expected cost without compromising the structural safety. The probabilistic design optimization allows us to consider the uncertainties as random variables in the optimization process. The probabilistic optimization can be formulated as

$$\begin{aligned} \min_{\mathbf{d}} \quad &: \quad C_T = C_c(\mathbf{d}) + P_f \; C_f \\ \text{Subject to} \quad &: \quad \begin{cases} \Pr\left[G_i\left(\mathbf{d}, \mathbf{X}\right) \leq 0\right] \leq P_f^T & i = 1, \ldots, m \\ h_j(\mathbf{d}) \leq 0 & j = m+1, \ldots, n \end{cases} \end{aligned} \tag{3}$$

where C_T is the total expected cost, defined in terms of initial cost C_c and failure cost C_f, P_f is the probability of failure, and P_f^T is the admissible failure probability. However, because of the difficulty of estimating the failure cost, especially when dealing with human lives, this objective function is usually replaced by the initial cost. The ultimate and serviceability constraints in deterministic optimization model [Eq. (2)] are transformed to probabilistic constraints, where the ith limit state function G_i is given in terms of design variables $\mathbf{d}$ and random variables $\mathbf{X}$. In the above model, the probabilistic constraints define the feasible domain, such as the failure probability P_f of the limit state G_i should be lower than the allowable probability P_f^T. This failure probability is given by the integral

$$Pr\left[G_i\left(\mathbf{d}, \mathbf{X}\right) \leq 0\right] = \int \ldots \int_{G_i(\mathbf{d}, \mathbf{X}) \leq 0} f_{\mathbf{X}}\left(\mathbf{x}\right) \; d\mathbf{x} \tag{4}$$

where $f_{\mathbf{X}}\left(\mathbf{x}\right)$ is the joint probability density function of the random variables $\mathbf{X}$. The evaluation of this integral can be done either by Monte Carlo simulation techniques or by moment methods, such as First Order Reliability Method (FORM). The moment methods are usually preferred because of their numerical efficiency and accurate evaluation of gradients. When FORM is used, the failure probability is approximated by $P_{f_i} = \Phi\left(-\beta_i\right)$, where $\Phi(\cdot)$ is the standard normal distribution. In this case, the probability constraints $\Pr\left[G_i\left(\mathbf{d}, \mathbf{X}\right) \leq 0\right] \leq P_f^T$ can be replaced by the reliability index constraints: $\beta_i\left(\mathbf{d}, \mathbf{X}\right) \geq \beta_i^T$; where $\beta_i\left(\mathbf{d}, \mathbf{X}\right)$ and β_i^T are respectively the reliability index and the target index, relative to the ith component.

For instance, the formulation in Eq. (3) is related only to component failure, without considering the system effect. However, real structures are made of a number of components, where the failure modes are not independent, even for statically determinate structures (especially for RC structures). The large majority of works in the system reliability analysis are based on the assumption of the weakest link theory; that is, the whole structure fails when any single component fails. However, the statistical correlation between the random variables and the interaction between the failure modes can lead to a system reliability significantly different from the one of the weakest component reliability [13]. Generally, component-based RBDO suffers from two difficulties: incapability of taking account for failure mode interactions and impossibility of optimal allocation of the target probability for each component. For these reasons, it is not interesting to separately optimize each component without controlling the other failure conditions, as the interactions modify the system cost and safety.

4. SYSTEM RELIABILITY-BASED DESIGN OPTIMIZATION (SRBDO)

For structural systems, the RBDO is formulated as the minimization of the cost function, by satisfying the system target reliability index β_{sys}^{T} as well as the target indexes for the components β_{i}^{T}; the last condition is added to ensure a lower bound for component reliabilities. The system RBDO is thus given in the form

$$\min_{\mathbf{d}} : \; C_c(\mathbf{d})$$
$$\text{Subject to : } \begin{cases} \beta_{sys}\left(\beta, \rho\right) \geq \beta_{sys}^{T} & \\ \beta_i\left(\mathbf{d}, \mathbf{x}\right) \geq \beta_i^{T} & i = 1, ..., m \\ h_j(\mathbf{d}) \leq 0 & j = m+1, ..., n \end{cases} \tag{5}$$

where $\beta_i\left(\mathbf{d}, \mathbf{x}\right)$ and $\beta_{sys}\left(\beta, \rho\right)$ are respectively the reliability indexes for the ith component and for the system, β_i^T and β_{sys}^T are respectively the target reliability indexes for the ith component and for the system, and $h_j(\mathbf{d})$ are deterministic structural constraints.

The component reliability indexes $\beta_i\left(\mathbf{d}, \mathbf{X}\right)$ are evaluated by using FORM, where the limit state G_i is approximated by a tangent hyperplane at the most probable failure point (MPFP) [6], which is found by solving the constrained optimization problem in the normalized space:

$$\min_{\mathbf{u}} \|\mathbf{u}\|$$
$$\text{Subject to} \quad : \quad G_i(\mathbf{u}) \leq 0 \tag{6}$$

The normalized random variables $\mathbf{u}$ are obtained by probability transformation of the random variables $\mathbf{x}$. The solution of the above expression leads to the most probable failure point (MPFP) $\mathbf{u}^*$, and the reliability index $\beta_i\left(\mathbf{d}\right) = \|\mathbf{u}^*\|_i$.

The system reliability index is described through the combination of component reliabilities, which can be given by FORM [9]. For series and parallel systems, the failure probability is computed through the union or the intersection of individual failure events:

$$\begin{aligned} p_{f_{sys}} &= P\left[\bigcup G_i \leq 0\right] = 1 - \Phi_m\left(\beta, \rho\right) && \text{for series system} \\ p_{f_{sys}} &= P\left[\bigcap G_i \leq 0\right] = \Phi_m\left(-\beta, \rho\right) && \text{for parallel system} \end{aligned} \tag{7}$$

where β is the vector of the component reliability indexes, ρ is the correlation matrix between failure modes, and $\Phi_m(.)$ is the multivariate cumulative normal probability function given by

$$\Phi_m\left(\beta, \rho\right) = \int_{-\infty}^{\beta_1} \dots \int_{-\infty}^{\beta_m} \frac{1}{\left(2\pi\right)^{m/2}\sqrt{\det\left[\rho\right]}} e^{-(1/2)\hat{u}^t \, \rho \, \hat{u}} d\beta_1 \dots d\beta_m \tag{8}$$

where $\hat{u}$ is a vector of correlated normal variables with correlation matrix ρ. The numerical integration of Eq. (8) is impractical when the number of failure modes is large $(m \geq 5)$. However, several approaches have been developed to evaluate the integral in Eq. (8), which can be classified into three categories: (a) bounding methods where practical expressions give the lower and upper bounds of the system failure probability [16]; (b) approximation methods, where Eq. (8) is evaluated by series expansion [9]; (c) sampling methods, where the multi-normal integration is evaluated by using efficient sampling method, such as sequential conditioned importance sampling algorithms [17]. In this work, Ditlevsen's bounds [16] are used to compute the system reliability index.

5. OPTIMAL SYSTEM SAFETY FACTORS FOR DESIGN OPTIMIZATION

The classical formulation of RBDO in Eq. (5) is still impractical for designing complex structures due to the different levels of nested procedures. Mainly, two optimization loops are nested: the outer loop consists in searching for the optimal design and the inner loop consists in performing the reliability analysis; both loops require repeated evaluations of the limit state functions, including finite element analyses.

In addition, the integration of component and system reliability constraints in the same formulation [Eq. (5)] leads to redundant definitions of constraints, due to the implicit relationship between component and system reliabilities. From the numerical point of view, when the system constraint is active, the component constraints become useless; this means that most of component constraints are inactive, which brings about the delicate problem of the assurance of optimal design. In this case, the system target reliability is satisfied while the solution is not optimum, but rather over-designed because some components may have high reliabilities [13]. Another drawback of this formulation concerns the evaluation of the system reliability gradient with respect to the design variables, which is generally carried out by finite difference. This computation is not only expensive but also inaccurate due to system reliability approximations. These difficulties lead to instability problems and therefore convergence cannot be guaranteed [18].

Recently, Du and Chen [14] have proposed an interesting formulation of RBDO, called SORA (sequential optimization and reliability assessment), where the probability measure is converted to a performance measure by solving an inverse reliability problem [19]. The performance measure computation is decoupled from the optimization procedure. Nevertheless, system reliability has never been integrated into the SORA method.

The proposed method aims to achieve the three following goals: (a) efficiency and robustness, (b) integration of the system target reliability, and (c) simplicity of implementation. To achieve efficiency, the method is based on the decoupled approach, where reliability analysis allows us to update the safety factors to be introduced in the design optimization procedure. The RBDO problem defined in Eq. (3) is converted into a sequence of DDO cycles [Eq. (2)] based on the safety factor concept (each cycle formed of several iterations until convergence). In addition, the system failure should be considered because the system safety can be lower than the component reliabilities. For this reason, the updating of the safety factors should be rational and balanced, in order to take account for system and component reliabilities. The simplicity of implementation requires that any general optimization procedure should be able to solve the RBDO problem. The proposed method is organized into five main steps:

1. The initial values of partial safety factors are chosen according to the codes of practice, such as Eurocodes or ACI.

2. The DDO cycle is carried out [Eq. (2)] by using the current safety factors, which are updated in each cycle.

3. At the convergence of DDO, the reliability analysis is performed for the optimal solution obtained in Step 1. Then, the system reliability index is evaluated by Eq. (7).

4. If the required system reliability is reached, and the solution cannot be improved, the procedure stops. Otherwise, the target component reliabilities are updated by iterative procedure, as indicated in Section 5.1. This procedure consists in finding the optimal target reliabilities for components, allowing guarantee of the required system reliability.

5. The safety factors are then calibrated on the basis of the optimal target reliabilities obtained in the previous step. That is the reason why these factors are called the *Optimal System Safety Factors* (OSSF). They are then used in the following cycle of DDO.

6. Steps 2 to 5 are repeated until convergence to the optimal cost and the system safety requirement.

In this scheme, the procedure of searching for optimal target reliabilities and the calibration of OSSF are decoupled from the optimization process, in order to avoid the nested loops of conventional RBDO formulation. Moreover, the use of the deterministic optimization leads to efficient and robust convergence with inexpensive computational effort. The procedure can thus be easily implemented in finite element software.

5.1 Updating the Target Component Reliabilities

The system reliability is given by the combination of component failure modes, through logical operators (i.e. AND/OR operators). In such a combination, the system reliability depends mainly on the modes with high failure probabilities.

It is however possible to reach the same system target reliability through infinite ways of setting the component targets. Mainly, the target reliability indexes of dominant modes have to be adjusted. For optimal design, the component target allocations should be performed to minimize the structural cost without altering the system reliability level.

The same system reliability can be met by infinite ways of component reliability combinations. The updating procedure consists of defining an appropriate objective function for the optimal reliability allocations. A reasonable goal consists of pulling down the component reliabilities to the level of the target system safety, through the least square minimization of the gap between the component and the target levels. The sensitive components are affected by weighting coefficients. This condition has the advantage of preventing any over-design of structural components, because a component is penalized when it has high reliability with no significant contribution in the system reliability. In this way, the procedure of searching for the optimal target reliabilities is written in the form

$$\begin{aligned} \min_{\beta^T} \quad &: \quad \sum_{i=1}^{m} \omega_i \, \left(\beta_i - \beta_i^T\right)^2 \\ \text{Subject to} \quad &: \quad \beta_{sys}\left(\beta^T, \rho\right) \; \geq \; \beta_{sys}^T \end{aligned} \tag{9}$$

where the target reliability indexes β_i^T are themselves the optimization parameters (β^T is the vector with components β_i^T) and ρ is the matrix of the limit state correlation coefficients. The optimal solution of problem (9) corresponds to the best quadratic fitting between the failure mode reliability indexes β_i and the corresponding target indexes β_i^T under the constraint of satisfying the system target; this constraint is always active at the optimal solution (i.e., $\beta_{sys}\left(\beta^T, \rho\right) = \beta_{sys}^T$). The weighting coefficients ω_i are the key to reach the optimal calibration, as they should take account safety and cost balancing. In this work, these coefficients have been defined in terms of the cost sensitivity regarding the changes in the safety margin (i.e., limit state function):

$$\omega_i \; = \; \sqrt{\frac{\partial C_c / \partial G_i\left(\mathbf{d}, \mathbf{x}_k, \gamma\right)}{\left\| \partial C_c / \partial G_i\left(\mathbf{d}, \mathbf{x}_k, \gamma\right) \right\|}} \qquad \text{with} \qquad \frac{\partial C_c}{\partial G_i\left(\mathbf{d}, \mathbf{x}_k, \gamma\right)} = \sum_{j=1}^{n} \frac{\partial C_c / \partial d_j}{\partial G_i\left(\mathbf{d}, \mathbf{x}_k, \gamma\right) / \partial d_j}$$

In this formulation, higher weights are given for the limit states with high influence on the system performance. As the derivatives of the structural cost and the limit state function, $\partial C_c / \partial d_j$ and $\partial G_i / \partial d_j$, are already computed in the deterministic loop, there is no additional computation effort to evaluate the weights ω_i. Although problem (9) can be solved by any general optimization algorithm, sequential quadratic programming [20] is chosen in this work, where the derivatives of the system reliability $\partial \beta_{sys} / \partial \beta_i^T$ are accurately computed by finite difference, as no mechanical analysis is involved to link system and component reliability indexes. Moreover, this task requires negligible computational time, because no additional Finite Element Analysis is performed.

5.2 Calibration of Optimal System Safety Factors

The proposed procedure is based on the calibration of the safety factors, which would be introduced in the deterministic optimization. After determining the optimal reliability indexes β_i^T, as described in the above section, the safety factors γ_{ij} have to be set for each variable u_j in the limit state function of the ith component $G_i(u_j)$. In other words, a full set of partial factors is related to each limit state. Contrary to the classical design concept, a matrix of partial factors is defined for random variables with respect to different failure modes, rather than the same factors for all the limit states. As $G_i(u_j)$ may be linear or nonlinear, the two cases should be considered.

The safety factors can be accurately computed directly by using the target index β_i^T and the direction cosines $\alpha_{ij} = \partial G_i / \partial u_j |_{\mathbf{u}^*}$ at the most probable failure point $\mathbf{u}^*$, provided by FORM. As shown in Fig. 1, the vector of direction cosines remains the same when the reliability index β is at the neighborhood of the target reliability level β^T. The calibration rule in this case is simply expressed by

$$\gamma_{ij} = \frac{F_{X_j}^{-1}\left[\Phi\left(\beta_i^T \alpha_{ij}\right)\right]}{x_{k_j}} \tag{10}$$

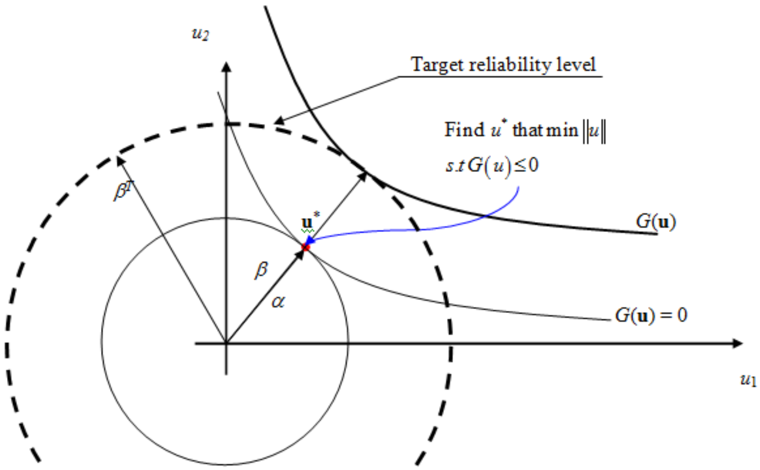

FIG. 1: Target reliability level and reliability index.

where α_{ij} is the direction cosine of the ith limit state with respect to the jth random variable, x_{k_j} and $F_{X_j}^{-1}(\cdot)$ are respectively the characteristic value and the inverse cumulative probability function of the jth random variable, and β_i^T is the optimal target reliability of the ith limit state (or component).

6. NUMERICAL RESULTS

The proposed RBDO approach is implemented in a computer program using Matlab^{TM} environment, which contains a very useful optimization toolbox. The comparison is made with other DDO and RBDO approaches. The application concerns a RC girder with T cross section.

6.1 Reinforced Concrete Girder

The goal of this example is to optimize the RC highway bridge girder shown in Fig. 2, where the target system reliability is given by $\beta_{sys}^T = 3.8$. Although the deck width is 3 m, only a part $b_f = 1$ m is effective for the T cross section supporting the bending moment, according to design codes. The following optimization methods are applied:

a. Deterministic Design Optimization (DDO) on the basis of the safety factors prescribed by the Eurocode 2.

b. RBDO using the classical approach [Eq. (5)], where the system and component reliability targets are set to $\beta_{sys}^T = \beta_i^T = 3.8$.

c. The herein proposed OSSF-RBDO, where the system target is: $\beta_{sys}^T = 3.8$.

The design variables are given by $\mathbf{d} = \{h_f, h_w, b_w, A_{s1}, A_{s2}\}$, where $A_{s1},\ A_{s2}$ are the tensile reinforcement steel areas, as showed in Fig. 3, h_f is the deck thickness, and b_w and h_w are respectively the web width and height. The random variables are $\mathbf{X} = \{P_1, P_2, Q_1, Q_2, f_c, f_y, E_s, \rho_{rc}\}$, where P_1, P_2 are the dead loads, excluding the girder own weight, Q_1, Q_2 are the live loads, f_y is the steel yield strength, f_c is the compressive concrete strength, ρ_{rc} is the RC unit weight, and E_s is the steel Young's modulus. For all the optimization methods, the initial design variables are taken as: $h_w = 100$ cm, $b_w = 60$ cm, $h_f = 40$ cm, $A_{s1} = 75.17$ cm^2, and $A_{s2} = 32.17$ cm^2. The random variables are considered independent with distributions and parameters indicated in Table 1. The system reliability corresponds to a series system with two failure modes, defined by the two ultimate limit states, corresponding to positive and negative bending moments:

$$\begin{aligned} G_1 &= M^+ - M_{u1} \le 0 \\ G_2 &= M^- - M_{u2} \le 0 \end{aligned} \tag{11}$$

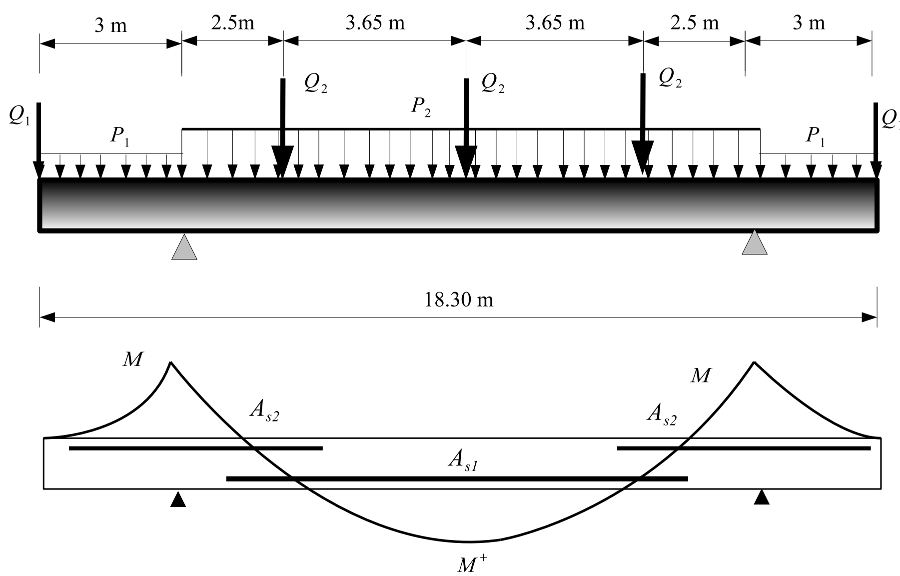

FIG. 2: RC girder and load configuration.

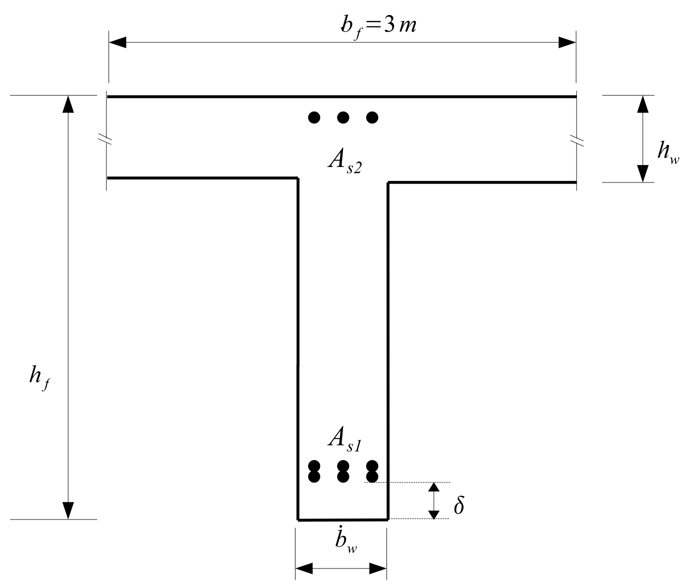

FIG. 3: RC girder cross section.

where M^+ is the positive bending moment at mid-span and M^- is the negative bending moment at the supports. The resisting moments at the two cross sections M_{u1} and M_{u2} are computed by appropriate integration of the stresses over the cross section, using the constitutive laws of concrete and steel given by the Eurocode 2.

The design optimization aims at minimizing the cost function defined by the sum of concrete, steel, and formwork costs:

$$C_{girder} = \underbrace{b_w L_g \left(h_f - h_w\right) c_{cr}}_{C_{cr}} + \underbrace{\left(A_{s1} L_{As1} + 2 A_{s2} L_{As2}\right) \rho_s c_{st}}_{C_{st}} + \underbrace{\left[2\left(h_f - h_w\right) + b_w\right] L_g c_{fw}}_{C_{fw}}$$

TABLE 1: Statistical data of random variables

Description	Variable	Characteristic value	Mean	C.O.V	Distribution
Dead load	Q_1/Q_2 (kN/m^2)	20.96/34.93 (95%)	18/30	0.10	Normal
Live load	P_1/P_2 (kN)	35.28/91.72 (95%)	25/65	0.25	Normal
Strengths	f_y/f_c (MPa)	500/30 (5%)	592/39	0.10/0.15	Lognormal
Young Modulus	E_s (GPa)	200 (50%)	200	0.05	Lognormal
RC weight unit	ρ_{RC} (kN/m^3)	25 (50%)	25	0.10	Lognormal

where L_g is the total beam length, $L_{A_{s1}}$ and $L_{A_{s2}}$ are respectively the lengths of steel bars for positive and negative moments, $\rho_s = 7850$ kg/m^3 is the unit weight of steel, $c_{cr} = 150.6$ €/m^3 is the concrete cost, $c_{st} = 1.46$€/kg is the steel cost, and $C_{fw} = 47$ €/m^2 is the formwork cost.

The DDO uses the safety factors in Eurocode 2, leading to the design values $f_{cd} = f_{ck}/1.5$ for concrete and $f_{yd} = f_{yk}/1.15$ for steel, where f_{ck} and f_{yk} are respectively the 5% quantiles (i.e., characteristic values). The maximum bending moments are computed by the fundamental combinations of applied actions. In classical RBDO, the loadings and strengths are directly considered as random (i.e., partial factors are not applied). In the proposed RBDO, the principle of safety factors is maintained but with optimal updating at each optimization cycle. Moreover, each limit state involved in the design introduces a safety factor for each random variable (i.e., a vector of safety factors is defined for each limit state).

Table 2 indicates the optimal solutions corresponding to different optimization methods and Table 3 compares the observed performance. It is clearly seen that deterministic optimization leads to a costly and over-designed system, where the optimal design is highly reliable regarding the target. Concerning RBDO methods, the classical and the proposed methods satisfy the system reliability requirement. However, OSSF-RBDO leads to slightly lower optimal cost, compared to classical RBDO. Knowing the cost sensitivity with respect to mid-span steels, the performance of the proposed updating procedure is clearly shown as the index of the limit state G_1 is pulled down from 4.02 to 3.88, in order to reduce the cost. The proposed approach searches for the optimal target reliabilities that verify the system target and calibrates the safety factors through optimal compromise of cost and safety.

From the numerical point of view, Table 3 indicates the high performance of the proposed method, which requires 513 evaluations of the mechanical model (G-eval), compared to 11,952 for classical RBDO, which means that the computation time is divided by a number close to 24. This reduction is extremely important in highly consuming finite element analysis of large-scale structures. A comparison of the numerical performances of the OSSF-RBDO with those of classical RBDO method and DDO approach is illustrated in Fig. 4. The proposed procedure requires less computation time, fewer iterations, and a smaller number of mechanical calls, because it is based on the DDO procedure, already known to be numerically efficient and robust. Figure 5 shows the cost history during the iterations, as the initial safety factor values are taken from the Eurocode 2, the proposed approach works as the deterministic optimization until the end of the first cycle (corresponding to the 7th iteration). The safety factors are then updated

TABLE 2: Comparison of the optimal design points

Method	h_f (cm)	h_w (cm)	b_w (cm)	A_{s1} (cm^2)	A_{s2} (cm^2)
DDO	74.7	20.8	34.1	93.2	32.1
OSSF-RBDO	62.1	17.3	28.3	80.3	26.8
Classical RBDO	63.7	17.8	30.3	79.8	25.1

TABLE 3: Numerical results for the girder design optimization

Method	Cos (k€)	β_{sys}	β_{G_1}	β_{G_2}	Iterations	CPU (s)	f-eval	G-eval
DDO	3.36	6.49	6.49	7.19	8	10	77	154
OSSF-RBDO	2.76	3.80	3.88	4.10	12 in 3 cycles	21	90	513
Classical RBDO	2.81	3.80	4.02	3.92	24	480	216	11952

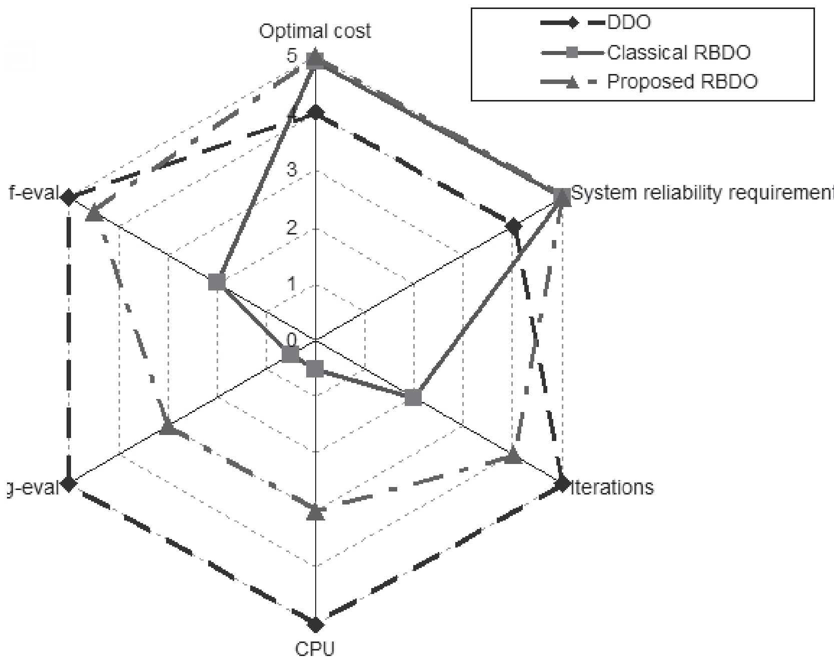

FIG. 4: Numerical performances.

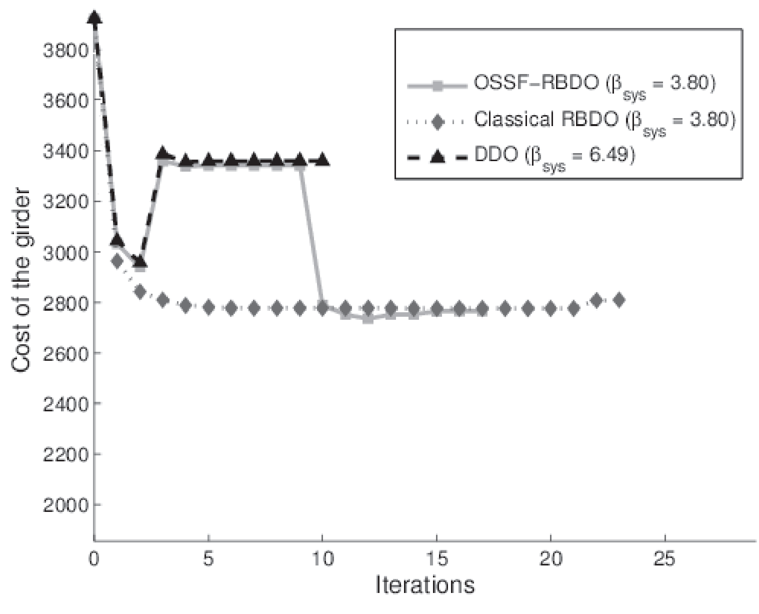

FIG. 5: Structural cost history.

in order to reach the optimal design, satisfying the target system safety. Table 4 gives the OSSF with respect to the median values, defined by $\gamma_d = X_d/X_{50\%}$ for loading variables and $\gamma_d = X_{50\%}/X_d$ for strength variables; they are called herein the *median-based factors*. The use of the characteristic value equal to the median is more suitable from the algorithmic point of view than the quantile at 5% or 95%.

TABLE 4: Partial safety factors and optimal system safety factors

γ_d		P_1	P_2	Q_1	Q_2	f_c	f_y	E	ρ_{RC}
DDO:	G_1	1.16	1.57	1.48	2.11	1.93	1.36	1	1.35
	G_2	1.57	1.16	2.11	1.48	1.93	1.36	1	1.35
OSSF:	G_1	0.96	1.26	0.81	1.08	1.08	1.31	1.01	1.02
	G_2	1.12	1	1.63	1	1.10	1.34	1.01	1.02

7. CONCLUSION

This work presents an efficient RBDO methodology, where the RBDO problem is transformed into a number of cycles of conventional deterministic optimization with optimally calibrated safety factors. At each cycle, system reliability analysis is performed in order to update the partial safety factors with respect to the considered limit states. These factors are then used in the next cycle of deterministic optimization. The proposed approach is very attractive because of its simple implementation in any general purpose optimization and finite element software, where the probabilistic analysis is decoupled from the optimization process, in addition to the use of the safety factor concept, which is familiar for designers. As the updated safety factors are computed from probabilistic analysis of the system, they represent the link between deterministic optimization and reliability analysis. The safety factors are calibrated on the basis of optimal setting of the component target reliabilities, allowing to fit the global system target.

The robustness and the efficiency of the proposed method are shown in the RC application, where the numerical performance is equivalent to DDO. Compared to conventional RBDO, the proposed method is much more efficient and presents the advantage of integrating the system reliability for optimal design.

The proposed method can be easily applied to any kind of structural systems without any loss of generality. Besides, the method has the interest of leading to balanced design, where component and system requirements are simultaneously fulfilled.

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