

## NOVAK ZUBER AND THE DRIFT FLUX MODEL

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I first heard of Novak Zuber when I was working on my MS at MIT during 1957–1959 and he was working on publishing the results of his Ph.D. thesis at the University of California, Los Angeles (UCLA). I am unsure if I met him at that time. If I did, it was only for a brief handshake, but I remember that his 1958 ASME paper (Zuber, 1958) “The Stability of Boiling Heat Transfer” created quite a stir and I was aware of it. I also learned of some related reports he had written for the Atomic Energy Commission (Zuber and Tribus, 1958; Zuber, 1959).

In 1961 I had moved to England, but I was in Boulder, Colorado, to present a paper (Wallis, 1961a) at the ASME International Heat Transfer Conference being held there. Novak presented his work (Zuber et al., 1961) on the hydrodynamics of pool boiling critical heat flux, in which he had derived the empirical result of Kutateladze (1951) using an idealized two-fluid model. There was quite a controversy over his paper and I stood up to defend him. I wrote a comment on his paper (Wallis, 1961c), deducing some of his results by alternative methods that were explained more fully in a later publication (Wallis, 1962b).

Novak’s supervisor at UCLA, Myron Tribus, was also there. We were talking and I learned that Myron was going to Dartmouth to be the Dean of the Thayer School of Engineering. He had creative ideas about developing the curriculum along the lines of engineering science that Boelter had pioneered at UCLA and that had already been partially implemented at Dartmouth. His ideas of teaching the basics of engineering methods in a unified way, without introducing them separately in individual disciplines, appealed to me very much. I had thought that it was unnecessary to teach very similar mathematical approaches in courses with separate labels such as electrical, mechanical, fluid dynamics, etc. He also intended to expand the curriculum to include Ph.D. and Doctor of Engineering degree programs. He would give priority to new faculty who would develop these ideas.

Another motive that Myron declared in suggesting that I come to Dartmouth, was to bring me closer to Novak, so that we could work together to develop a sound theoretical

basis for the expanding fields of two-phase flow and heat transfer. I agreed to come. In the summer of 1962 I was hired to work alongside Novak in a temporary position in the General Engineering Laboratory at General Electric in Schenectady, New York, before moving to teach at Dartmouth in the fall.

During that summer, and after I had moved to Dartmouth, we prepared some internal reports and drafts for a post-graduate summer course. Novak had conceived of the idea of adapting the treatment of the equimolecular counter-diffusion of gases to two-phase flow that was developed by Bird et al. (1960, Chapter 16). The relative motion would be treated like the diffusion flux, and it appeared in that form in earlier drafts. This diffusion flux resembled the characteristic relative velocity that I had developed in my Ph.D. thesis in 1961 and used in a few publications (Wallis, 1961a, 1962a,c). It successfully represented the results of bubbling water through mercury obtained by Kutateladze and Moskvicheva (1959), when the phases were inverted and the mercury became the dispersed phase. This fluidization was described by the same formula that Novak had developed (Zuber, 1958) to describe the critical (maximum) boiling heat flux based on a countercurrent flow of columns of each phase. Although often empirically successful, neither of these idealized models appears to closely resemble what is actually observed at the critical heat flux condition during pool boiling.

In Novak's paper "On the dispersed two-phase flow in the laminar flow regime" (Zuber, 1964), he reviewed a broad range of literature, presented expressions for the relative motion between the phases, and applied them to topics such as sedimentation, fluidization, flooding, and kinematic waves, much as I had done using different nomenclature. He uses the term flux density for the volumetric flow rate per unit area, but still retains the idea of diffusion.

We discussed how best to represent these formulations. It seemed inappropriate to invoke diffusion as the relative motion in the applications we were interested in, such as vertical flow in nuclear reactor coolant channels, which was not primarily driven by diffusion but by the action of gravity on the different densities of the phases. The word drift almost certainly came from Novak, as he was more familiar with the literature and had probably read the classic paper by Lighthill (1956), who was concerned with the way that an object moving through a fluid tends to carry along some fluid with it—a phenomenon linked to the concept of added mass. There is relative motion in this case that drift does not describe, it only describes its consequences. Nevertheless, we decided to replace the word diffusion with what we called drift when describing the relative motion between the phases. The average volumetric or mass flow rates per unit area of each phase would be called fluxes instead of the rather confusing term superficial velocity that was current in the chemical engineering literature and had been used in my work up to then. The use of velocity would be confined to the description of the average velocity of each phase and to the relative velocities of various sorts. The first use of the term drift velocity that I found in Novak's work appears in the seminal paper, "Average Volumetric Concentration in Two-phase Systems" (Zuber and Findlay, 1965).

The key development is a simple expression for the velocity of the vapor phase relative to a weighted average of the total volumetric flux, but there is no mention of the term drift flux in that paper.

In 1964, Novak and I discussed how best to represent these phenomena and what definitions and nomenclature to use. The definition that I had used of a characteristic relative velocity was as follows:

$$V_{cd} = (1 - R) V_d - R V_c \quad (1)$$

where subscripts  $c$  and  $d$  denote the continuous and discontinuous phases, respectively;  $R$  denotes the volumetric concentration of the discontinuous phase; and  $V$  denotes the superficial velocity, became a “drift flux:”

$$j_{21} = (1 - \alpha) j_2 - \alpha j_1 \quad (2)$$

The “Drift Flux Model” is the title of Chapter 4 in my book (Wallis, 1969), much of which evolved from summer courses given at Dartmouth during the 1960’s in which Novak participated. I cannot identify any specific historical moment in which the particular term was conceived or came into use. The volumetric fluxes are denoted by  $j$ , the phases are denoted by subscripts 1 and 2, and the volumetric concentration of the dispersed phase is denoted by  $\alpha$ . We also defined drift velocities,  $v_2 - j$  and  $v_1 - j$ . The drift flux was related to these, and to the average relative velocity between the phases by

$$j_{21} = \alpha (v_2 - j) = - (1 - \alpha) (v_1 - j) = \alpha (1 - \alpha) (v_2 - v_1) \quad (3)$$

This represents the equal and opposite fluxes of the phases relative to the average total flux,  $j = j_1 + j_2$ . For instance, in an upward vertical flow of gas and liquid this relative volumetric exchange rate gives rise to a dissipation of mechanical energy per unit volume (a property often used in the chemical engineering literature to describe mixing and heat and mass transfer) of  $j_{21} g (\rho_f - \rho_g)$ . The drift flux turned out to be useful in describing and predicting many phenomena involving the one-dimensional (1D) vertical flow of dispersions, including transient behavior, as described in my Ph.D. thesis (Wallis, 1961b), Novak’s paper (Zuber, 1964), and in Chapters 4, 6, 8, 9, and 12 of my book (Wallis, 1969).

The simple 1D version of the drift flux theory is useful in representing flows that are essentially uniform across the duct and are driven by gravity acting on the different densities of the phases and countered by interphase hydrodynamic drag, such as sedimentation of powders in a test tube, draining of foam in a beer glass, or fluidization of particles in a chemical reactor. In many such cases it is possible to represent the drift flux as a function of the volumetric concentration of the dispersed phase by a relationship of the following form:

$$j_{21} = v_\infty \alpha (1 - \alpha)^n \quad (4)$$

where  $v_\infty$  is the velocity of an element of the dispersed phase in the limit of very low concentration.

When there is significant flow of a mixture relative to a duct, which leads to variations across the flow, such as a velocity profile for  $j$ , deviations from this simple approach occur. Novak (Zuber and Findlay, 1965) had the idea of averaging the local relationship between the flux of phase 2 and the drift flux,  $j_2 = j_{21} + \alpha j$ , across the duct, denoting averaged values by brackets, to obtain

$$\langle j_2 \rangle = \langle j_{21} \rangle + \langle \alpha j \rangle \quad (5)$$

He then related the term involving the average of the product in Eq. (5) to the product of averages by the following definition:

$$\langle \alpha j \rangle = C_0 \langle \alpha \rangle \langle j \rangle \quad (6)$$

Inserting Eq. (6) into Eq. (5) and dividing by the average void fraction led to

$$v_2 = \langle j_2 \rangle / \langle \alpha \rangle = \langle j_{21} \rangle / \langle \alpha \rangle + C_0 \langle j \rangle \quad (7)$$

where  $v_2$  is the average velocity of phase 2, defined from the flow rate of that phase per unit area and the average volumetric concentration, as in a 1D model. Here,  $\langle j \rangle$  is the overall volumetric flow rate divided by the area of the duct. The term involving the drift flux is not strictly derivable from a correlation such as in Eq. (4) unless the concentration distribution is known; however, it may be assumed to be some appropriate drift velocity  $v_0$  that might be correlated as a function of the average concentration. Then, we may drop the averaging signs and simply state that a reasonable approximate representation is

$$v_2 = v_0 + C_0 j \quad (8)$$

A physical interpretation of Eq. (8) is that phase 2 has a drift velocity relative to some effective overall motion that differs from the average by the factor  $C_0$ . For example, bubbles in a pipe that are larger than the boundary layer on the wall will tend to move relative to the velocity in the core of the flow, which in a turbulent flow is about 1.2 times the average value. The justification for Eq. (8) is that it successfully correlates data under such conditions, the value of  $C_0$  being typically in the range of 1–1.5, with 1.2 being a good approximation in many cases. The graphical relationship between  $v_2$  and  $j$  is empirically found to be close to linear (Zuber and Findlay, 1965). Ishii collaborated with Novak, who was his Ph.D. professor at New York University, to derive several correlations for  $v_0$ , particularly for nuclear reactor applications (Ishii et al., 1975, 1976; Ishii and Zuber, 1979). The expression for  $v_2$  in Eq. (8) is identical to that given by Nicklin et al. (1962) for the bubble velocity in vertical slug flow at high Reynolds numbers, and therefore the approach is useful in describing that regime as well. Thus,  $C_0$  was found to be 1.2, and  $v_0$  was the velocity of the bubble in stagnant liquid.

Equation (8) has become the basis for what is often called the drift flux model, which has been used to predict behavior in several parts of nuclear reactor cooling systems, particularly in relation to the vertical upflow in the passages cooling the core of a boiling water reactor when using commercial versions derived from the original Zuber and Findlay (1965) publication. If the flow rates of the water and steam are known, typically from an energy balance, Eq. (8) supplies enough information to compute  $v_2$ ,  $v_1$ , and  $\alpha$ . The drift flux itself is not used in the analysis, and the model might perhaps be better described as a vapor velocity model, which is the original form developed by Zuber and Findlay (1965).

Several theorists have developed elaborate computer codes for predicting the performance of entire nuclear cooling systems based on a two-fluid model in which the phases have different velocities and obey suitable approximate conservation laws, particularly those for mass, momentum, and energy. Equation (8) is invoked as a closure relationship with the term drift flux model often attached to it. For general applications, this appears to be an extension of the concept beyond its range of applicability because the relative motion of the phases is no longer always determined by the simple balance between gravity and interphase drag on bubbles that gave rise to Eq. (8). A different flow regime, such as drop-annular flow, involves additional phenomena, such as wall friction and entrainment, as does a rapidly accelerating flow in which inertia effects may dominate, or a flow in bends and more complex geometries. A drift flux may always be defined, as in Eq. (2), and sometimes a suitable vapor velocity can be correlated; however, this is less useful unless it describes some fundamental aspect of the relevant physics or simplifies the calculation procedures.

## REFERENCES

- Bird, R. B., Stewart, W. E., and Lightfoot, E. N., *Transport Phenomena*, New York: Wiley, 1960.
- Ishii, M., Chawla, T. C., and Zuber, N., Constitutive equation for vapor drift velocity in two-phase annular flow, *AIChE J.*, vol. **22**, no. 2, pp. 283–289, 1976.
- Ishii, M., Jones, O., and Zuber, N., Thermal nonequilibrium effects in drift flux model of 2-phase flow, *Trans. Am. Nucl. Soc.*, vol. **22**, pp. 263–264, 1975.
- Ishii, M. and Zuber, N., Drag coefficient and relative velocity in bubbly, droplet or particulate flows, *AIChE J.*, **25**, no. 5, pp. 843–855, 1979.
- Kutateladze, S. S., A hydrodynamic theory of changes in the boiling process under free convection, *Izv. Akad. Nauk. S.S.S.R. Otd. Tech. Nauk.*, vol. **4**, pp. 529–536, 1951.
- Kutateladze, S. S. and Moskvicheva, V. N., The relation between the hydrodynamics of a two-component layer and the theory of critical states in the mechanism of boiling, *Zh. Tech. Fiz.*, vol. **29**, no. 9, pp. 1135–1139, 1959.
- Lighthill, M. J., Drift, *J. Fluid Mech.*, vol. **1**, no. 01, pp. 31–53, 1956.
- Nicklin, D. J., Wilkes, J. O., and Davidson, J. F., Two phase flow in vertical tubes, *Trans. Inst. Chem. Eng.*, vol. **40**, p. 61, 1962.

- Wallis, G. B., Some hydrodynamic aspects of two-phase flow and boiling, *International Developments in Heat Transfer*, vol. **II**, pp. 319–340, New York: ASME, 1961a.
- Wallis, G. B. Two-phase flow phenomena associated with the boiling of liquids, Ph.D. Thesis, University of Cambridge, UK, 1961b.
- Wallis, G. B., Written discussion of the hydrodynamic crisis in pool boiling of saturated and subcooled liquids, *Proc. of Heat Transfer Conference*, Boulder, CO, 1961c.
- Wallis, G. B., A simplified one-dimensional representation of two-component vertical flow and its application to batch sedimentation, *Proc. of Institute of Chemical Engineers Symposium on the Interaction between Fluids and Particles*, London, pp. 9–16, 1962a.
- Wallis, G. B., Two-phase flow aspects of pool boiling from a horizontal surface, Paper No. 3, *Proc. of IMechE Symposium on Two-Phase Fluid Flow*, London, pp. 19–28, 1962b.
- Wallis, G. B., *One-Dimensional Two-Phase Flow*, New York: McGraw-Hill, 1969.
- Zuber, N., On the stability of boiling heat transfer, *Trans. ASME*, vol. **80**, p. 711, 1958.
- Zuber, N., Hydrodynamic aspects of boiling heat transfer (thesis), *AECU-4439*, Atomic Energy Commission, Oak Ridge, TN, 1959.
- Zuber, N., On the dispersed two-phase flow in the laminar regime, *Chem. Eng. Sci.*, vol. **19**, pp. 897–917, 1964.
- Zuber, N. and Findlay, J. A., Average volumetric concentration in two-phase flow systems, *J. Heat Transfer*, vol. **87**, no. 4, pp. 453–468, 1965.
- Zuber, N. and Tribus, M., Further remarks on the stability of boiling heat transfer, *AECU-3631*, Atomic Energy Commission, Oak Ridge, TN, 1958.
- Zuber, N., Tribus, M., and Westwater, J. W., The hydrodynamic crisis in pool boiling of saturated and subcooled liquids, Paper No. 27, *Proc. of International Heat Transfer Conference*, Boulder, CO, vol. **II**, pp. 230–234, 1961.