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Comments on "Natural Convection from a Vertical Plate in a Saturated Porous Medium: Nonequilibrium Theory" by A. A. Mohamad

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A recent article by Mohamad (2001) investigates exactly the same convective boundary layer flow as an article by Rees and Pop (2000), although it was impossible for the later author to have known about the slightly earlier investigation.

The primary reason for this note is to emphasize differences in the approaches taken by the respective authors and to underscore an essential aspect of the numerical study of convective boundary layer flows in porous media where local thermal nonequilibrium is significant.

First, we mention that the flow is governed by only two nondimensional parameters, H and γ (Rees and Pop 2000), rather than by three, Π, K_r and ϕ (Mohamad 2001). The parameters are related according to the formula

$$H = \Pi/\phi, \quad \gamma = K_r\phi/(1-\phi) \tag{1}$$

where ϕ is the porosity in the notation of Mohamad (2001). In fact, as pointed out in Rees and Pop (2000), it is even possible to eliminate H from the equations, thereby reducing the mathematical problem to one involving only one parameter.

Second, we have found that the imposed far-field boundary condition for the solid-phase temperature field in Mohamad (2001) can lead to inaccuracies, since the detail of the evolution of the temperature field near the leading edge is quite subtle. The literature is replete with examples of boundary layer flows that evolve into a double-layer structure at *large* distances from the leading edge, examples of which include the studies of Rees and Bassom (1996) and Rees (1997). Typically this means that an asymptotic analysis is required to resolve an increasingly thin near-wall layer as x becomes large, since the numerical analysis will become decreasingly able to resolve this sublayer. When thermal nonequilibrium effects are present, however, the solid phase temperature field has a considerably greater thickness near the *leading* edge than does the fluid phase, that is, $\eta \gg 1$ as opposed to $\eta = 0(1)$, where η is the usual similarity variable. In practice, then, it is essential to carry out an asymptotic analysis for small values of x in order to determine how the solid-phase temperature field varies.

The analysis of Rees and Pop (2000) gives the variation of ϕ (the solid-phase temperature field, in their notation) in the $\eta \gg 1$ region near the leading edge. This exponentially decaying profile was then converted into an equiva-

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lent boundary condition in order to maintain a reasonably sized computational domain. To be specific, ϕ satisfies the equation,

$$\phi'' = H\gamma x(\phi - \theta) \tag{2}$$

where primes denote derivatives with respect to η and θ is the fluid phase temperature field. The large-η boundary condition used by Rees and Pop (2000) is

$$\phi' + \sqrt{h\gamma x}\phi = 0 \tag{3}$$

whereas Mohamad (2001) uses

$$\phi = 0 \tag{4}$$

Equation (3) allows ϕ to exhibit exponential decay but does not constrain ϕ to be zero at the edge of the computational domain.

In Figure 1 we display the results of applying the boundary conditions given in (3) and (4) to the boundary layer problem, and the effect of different sizes of computational domain is depicted. A nonuniform grid in the x direction was used, and the surface rates of heat transfer for both phases are plotted against ξ where $\xi = \sqrt{x}$. A uniform grid of size $\delta\eta = 0.1$ was chosen for illustrative purposes, and we have used $\eta_{max} = 10$, 20, 40, and 80 for cases employing boundary condition (4), and $\eta_{max} \geq 20$ for boundary condition (3). We have also used $H = \gamma = 1$ as representative values of these parameters.

It is clear from Figure 1 that the evolution of the surface heat transfer of the fluid phase is hardly affected by the value of η_{max} when $\eta_{max} \geq 20$ when using boundary condition (4) for ϕ. But the surface heat transfer of the solid phase is strongly affected. The deviation from the correct behavior is most evident at $x = \xi = 0$, and this is because the solid phase, whose temperature satisfies the equation $\phi'' = 0$ there, has the solution

$$\phi = 1 - \eta / \eta_{max} \tag{5}$$

and therefore the surface rate of heat transfer is

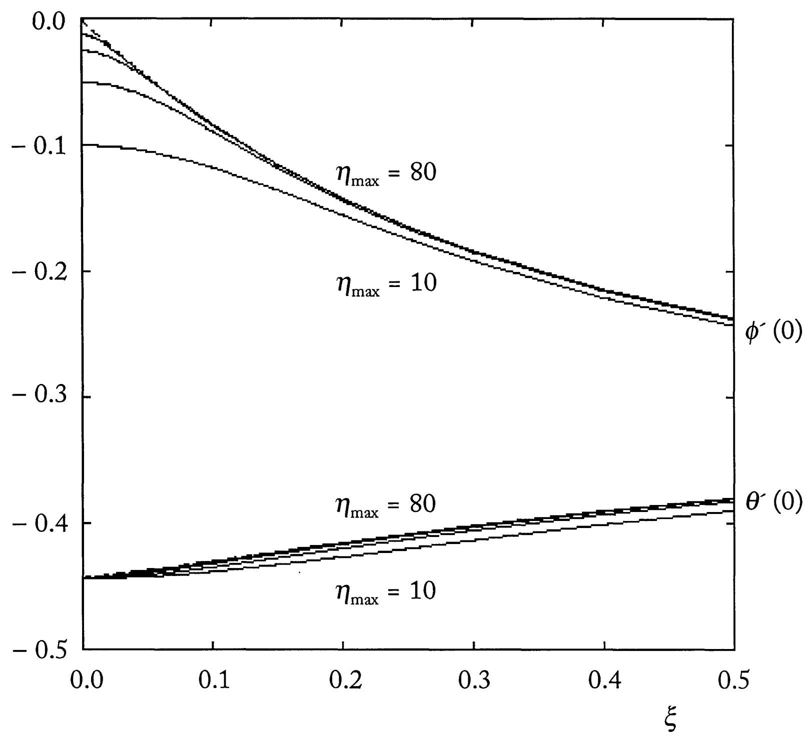

Figure 1. Evolution with $\xi(= x^{1/2})$ of the surface rate of heat transfer for both the fluid and solid phases (θ' and ϕ' at $\eta = 0$, respectively). The four continuous curves correspond to solutions of the governing equations subject to the boundary condition (4) for the following values of η_{max} :10, 20, 40, and 80. The dashed curve corresponds to the boundary condition (3) and $\eta_{max} \geq 20$.

$$\phi'(\eta = 0) = -1/\eta_{max} \quad (6)$$

This means that the numerical solution is highly dependent on the size of the computational domain when using boundary condition (4), although the distance in terms of ξ over which it deviates significantly from the correct solution diminishes as η_{max} increases.

On the other hand, the employment of boundary condition (3), which is shown as a dashed line for $\eta_{max} = 20$, gives rates of heat transfer that vary by at most 10^{-4} from the given curve when η_{max} is increased. Therefore boundary condition (3) yields solutions that are essentially independent of the size of the computational domain.

To conclude, it is necessary when considering local thermal nonequilibrium in porous medium boundary layer flows to undertake a detailed asymptotic analysis of the leading edge region in order to obtain boundary conditions for the solid-phase temperature field that are capable of describing accurately its behavior outside the computational domain. This technique has already been applied successfully in the article by Rees and Pop (1999). We also note its use in Rees and Vafai (1999), which deals with the Darcy-Brinkman convection from a vertical surface.

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